Uncertainty analysis of ANN based spectral analysis using Monte Carlo method

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**Abstract.** Uncertainty analysis of an Artificial Neural Network (ANN) based method for spectral analysis of asynchronously sampled signals is performed. Main uncertainty components contributions, jitter and quantization noise, are considered in order to obtain the signal amplitude and phase uncertainties using Monte Carlo method. The analysis performed identifies also uncertainties main contributions depending on parameters configurations. The analysis is performed simultaneously with the proposed method and two others: Discrete Fourier Transform (DFT) and Multiharmonic Sine Fitting Method (MSFM), in order to compare them in terms of uncertainty. Results show the proposed method has the same uncertainty as DFT for amplitude values and around double uncertainty in phase values.

**Keywords:** sine-fitting methods, spectral analysis, ADALINE, ANN, digital measurement, uncertainty, Monte-Carlo, DFT.

1. Introduction

The use of nonlinear electronic components connected to the electricity grid it is becoming more common in our daily life. These components add harmonic and inter-harmonic content to the electric signal, which results in a deterioration of the quality of the power supplied, an increase in losses and a decrease in the reliability of the whole system. In this situation, it is increasingly important to accurately determine the harmonic content of the power signals. For these reasons, several National Metrological Institutes (NMIs) have implemented methods to measure power in non-sinusoidal conditions [1] [2]. These new methods make use of the versatility of the digital techniques, especially considering the possibility of obtaining the spectral analysis of the signals of interest, and are based on the use of traditional algorithms, as DFT. These methods require synchronous sampling in order to be accurate. To overcome this problem, some authors [3] [4] have proposed the use of asynchronous sampling combined with the use of non-synchronous spectral analysis. In particular sine fitting methods [5] have been extended successfully to the multi-harmonic case [6].

Alternatively, a new method based on ANN was presented [7]. The method is based in the implementation of the multi-harmonic sine fitting algorithm [5] by mean of a multilayer perceptron neural network (MLP). The ANN method, as the sine fitting, has the advantage with respect to the traditional spectral analysis methods that it does not require synchronism between generation and sampling, which reduce the complexity of the hardware implementation. In comparison with the conventional multi-harmonic sine fitting method, the ANN method, has a simpler implementation and does not require special adjustments of coefficients for convergence, that depend on the type of signal being analyzed[6]. The propose method has been implemented in the Spanish electrical power primary standard, at Centro Español de Metrología [9] [10]. Additionally, this approach has the advantage to allows, with reduced complexity, modifications of the signal model to be analyzed. In this sense, this work was extended recently to obtain inter-harmonic components in [8].

At a primary level, the traceability of such sampling systems is a complex task due to the complexity of the tests and validations of involved algorithms. The main contribution of this work is the validation for sinusoidal signals of the ANN based spectral analysis method proposed in [7] by means of Monte Carlo method. In order to obtain useful results for real applications, parameters configurations used in laboratory will be used.

Following sections are structured as follows: in section 2 the system to be characterized is briefly presented; in section 3 the Monte Carlo method applied to the characterization of high precision measurement systems is introduced; methodology of the tests carried out is described in section 4 and results are reported in section 5; finally, section 6 exposes the conclusions of this work.

1. Network description

Let us consider a signal of interest, *y(t)*, stationary in the range of analysis, formed by *K* harmonic frequencies of the fundamental frequency, *fac*. The signal can be described mathematically in the terms of eq. (1), where *y(t)* is formed by the sum of the contributions of the DC component, the fundamental frequency, *fac*, and its multiples, *fk=k•fac, k ∈[2, K]*.

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The proposed ANN based method [7], MANNFM (Multiharmonic ANN Fitting Method), performs the spectral analysis of a steady state periodical signal composed by harmonics as described in equation (1). This is done by means of the multilayer neural network of Figure 1.

Input data to the ANN are the *N* time instants - *t[n], n [0, N-1]*- at which the samples of *y(t), y[n]*, are taken. The time vector, *t*, is scaled by the factor *2·π* in order to generate the arguments of cosines and sines of eq. (1).

The network implements the *K* order Fourier series synthesis equation. Briefly, Layer 1, implemented by means of an ADALINE neuron without bias and linear transfer function, has the object to implement the scale factor of the fundamental frequency, *fac*, in the arguments of the cosines and sines of the synthesis equation. The output of Layer 1 is connected in parallel to Layers 2 and 3, both of which are implemented by *K* ADALINE neurons with cosine and sine transfer functions respectively. The position of each neuron in the array of neurons determines its weight value, so that the *k* scale factor of each harmonic in the cosine and sine arguments is implemented by this weight. As a result, the input to the transfer functions of the neurons of Layers 2 and 3 are the completed arguments of the cosines and sines of the Fourier synthesis equation.



**Fig. 1.** The ANN architecture proposed [7].

Consequently, the input from the harmonic section to Layer *4* are the arrays *cos(2·π·k·fac·t), k∈ [1, K]* and *sin(2·π·k·fac·t) , k∈ [1, K]*. Layer *4* implements, finally, the sum of the synthesis equation, by means of *2·K* neurons with linear transfer functions. The weights of the neurons connected to Layer *2* are identified as the *Ak* coefficients of eq. (1), and the weights of the neurons connected to Layer *3* are identified as the *Bk* coefficients of eq. (1). In addition, Layer *4* includes the DC component of the Fourier synthesis series, *C*, by means of its bias value, which is available in this layer. Therefore, at this point, the output of Layer *4* is equivalent to the output of the eq. (1), *y(t)*, in the time instants *t[n], n ∈[0, N-1]*.

As mentioned above, this high precision algorithm has been implemented and tested in the measurements system of the Spanish NMI as part of the electrical power primary standard.

1. Propagation of distributions and simulation using the Monte Carlo method

As mentioned previously, due to the complexity of the MANNFM uncertainty evaluation, the MCM is used. In this section the basis of the MCM and its application to this purpose are presented. The Guide to the Expression of Uncertainty in Measurement (GUM) [11] provides the framework for uncertainty evaluation, which has three main stages: formulation, propagation and summarizing. At the *Formulation* stage the *N* input quantities, ***X*** *= (X1, …, XN)* upon which the output quantity, *Y,* depends, are determined and their probability distribution functions (PDF), are assigned. In the *Propagation* stage, the PDFs for the *Xi* are propagated through the measurement model, *Y= f(****X****)*, in order to obtain the PDF of the output *Y*. Finally, in the *Summarizing* stage, using de PDF obtained for *Y*, the expectation of *Y*, *y*, and its standard deviation, *u(y)*, are obtained[[1]](#footnote-1).

The propagation of distributions can be implemented analytically if a mathematical representation of the PDF for *Y* can be obtained. However, in practice this option is possible in simple cases and for more complex situations approximations based on Taylor series must be applied. When these approximations are not possible either (e.g., complex or nor linear models) numerical methods must be used.

The base of MCM [12], Figure 2, is to sampling the PDF for *Y*, obtaining *M* values *yr*, for *r =1, ..., M*. Each *yr* is obtained by sampling at random from each of the PDFs for *Xi*, and evaluating the model at the samples values so obtained. In case of several output variables [13], the foundation is the same but ***Y*** is a vector provided by a vector function or model ***f(X)***.



**Fig. 2.** MCM propagation of distributions for 3 independent input quantities

Consequently, in order to apply MCM, it is only necessary that *Y* can be formed from the values of *X*. Although Monte Carlo method is used as a mean to provide a numerical representation of the PDF for *Y*, it is also a simulation process that provides information about the input/output PDFs relationship of a system modeled as *f(X)*. So, the method can be used not only to obtain output uncertainties of a measurement, but also to simulate a system [14] and evaluate the impact of different parameters as well as the impact of the different uncertainty contributions in the output uncertainties. The second approach will be used in this work.

1. Methodology applied
   1. Simultaneous comparison of three methods

In order to compare MANNFM results with those obtained by other methods, the same tests have been applied to other two methods: DFT [15] and MSFM [6]. DFT can be considered as a reference, given that, in ideal conditions, provides the spectrum of the signal under analysis without error. When synchronous sampling is not possible, fitting methods are used [3] [5] in order to avoid DFT errors due to spectral leakage. As MANNFM, MSFM is a fitting method, but solved in a conventional way, solving a non-linear equation system iteratively.

As can be observed in Figure 3, for comparison purposes the same signal, *x(t)*, will be sampled and applied to the three methods. The DFT will process synchronous samples of *x(t)*, *xs[n]*, while MSFM and MANNFM will process asynchronous samples of *x(t)*, *xa[n]*, obtained in a set of time instants *ta[n]*, where *ta[n]=t[n]·(1+ ξa)*, being *ξa* the error of synchronism (relative) between generation and sampling.



**Fig. 3.** Scheme for the comparison of the three methods

* 1. Uncertainties contributions considered

Two uncertainty components will be considered in this work, jitter contributing to time uncertainty of each sampled point, the sampling jitter; and quantization contributing to a voltage uncertainty of each sampled point. . Both of them have been theoretically studied for DFT [16] [17].

Regarding jitter, a rectangular distribution has been considered. Taking into account the specifications of usual laboratory instruments, two jitter values (maximum) will be contemplated: 3·10-12 s and 1·10-7 s. The first value is the jitter specification for the National Instruments 5922 digitizer, and the second one is the jitter specifications for the Keysight 3458A digital multimeter used in [9] [10].

Concerning the quantization, it has been simulated taken into account three usual laboratory resolutions or number of bits, *Nbits,* values: *16*, *18* and *21* bits, which correspond again with the HP 3458A resolution derived from the aperture time ranges configured for high, medium and low sampling frequency respectively. Regarding the full scale parameter, *1.2* V has been used, which is the value for the *1* V range of the HP 3458A in digital operation mode.

In order to evaluate independent and jointly the impact of both contributions to the total uncertainty of the three methods, the scheme presented in Figure 3 will be simulated with three different sampling situations: (i) samples affected only by quantification; (ii) samples affected only by jitter; and (iii) samples affected by quantification and jitter. Terms *q, j* and *jq* respectively will be used in reference to signals and results related to this three sampling situations.

* 1. Parameters values considered

Several parameters values (according to real values used in laboratory) of the signal to be sampled, have been considered in order to evaluate their impact to the estimated uncertainty. A sinusoidal signal has been consider, with fundamental frequency, *fac*, *53* Hz and amplitude *1* V. Concerning to the phase of the sampled signal, uniformly distributed random values between ±π radians have been generated.

Regarding the ratio *R=fs/fac*, being *fs* the sampling frequency, three values have been selected, around *24*, *124* and *261*, related to low, medium and high sampling frequencies, respectively.

Concerning the number of samples, *N*, it has to be considered that (i) the Keysight 3458 internal memory maximum capability (for maximum precision) is *37888* samples and (ii) *N* and *M* should not have common factors in order to obtain maximum information of the sampled signal. Consequently, three *N* values (*1311*, *7073* and *15457*), related to low, medium and high number of samples have been selected and used in each ratio value. Being *M* the number of periods involved in the measurement, *R* can be also expressed as *R=N/M*. So, variations of *N* keeping *R* constant implies also to vary *M*. Table 1 shows the values of *R*, *N* and *M* considered. In order to assure the DFT conditions, for *N* and *M* integer values have been selected (which imply light variations of the ratio *R* obtained).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **N** | | **M** | **R** | **fs** |
| H | 15457 | 59 | 261.983 | H |
| M | 7073 | 27 | 261.963 |
| L | 1311 | 5 | 262.200 |
| H | 15457 | 125 | 123.656 | M |
| M | 7073 | 57 | 124.088 |
| L | 1311 | 11 | 119.182 |
| H | 15457 | 625 | 24.731 | L |
| M | 7073 | 286 | 24.731 |
| L | 1311 | 53 | 24.736 |

**Table 1.** Sampling frequency parameters values considered to estimate the uncertainty of the system.

Finally, about the error of synchronism, *ξa*, random values up to *±20%,* uniformly distributed, have been considered. The Interpolated FFT (IpFFT) [18] algorithm has been used in order to obtain initial estimation of the fundamental frequency for the asynchronous methods. No convergence problems have been detected in any simulation.

* 1. Computational hardware used

In order to perform the MCM in the different configurations established, large computational resources are required. For this work, resources provided by the University of Málaga‘s supercomputing center (UMSC) have been used. The computational resources are based in the Cluster Intel E5-2670, with shared memory machines with 2 TB of RAM each and AMD Opteron 6176 and ESX virtualization clusters.

1. Results

All tests have been performed in the same way: with the selected parameters and the uncertainty contributions defined in section 4.2, 106 trials of the MCM has been performed over the three methods (as described in section 4.1) in order to obtain a numerical representation of their PDF outputs.

All the obtained PDFs are Gaussian. The values have been expressed in deviations to nominal (absolute) and those deviations have been fitted to a normal PDFs by means of Matlab© *normfit* function, obtaining their mean and standard deviation, σ. For each output of each method, mean represents the deviation to nominal and standard deviation its uncertainty for a confidence level of 62.8%. In next subsections we will present the most significant results of the tests performed, in terms of mean and standard deviation.

### Methods comparison

A first test was performed in order to evaluate similarities between methods outputs and, at the same time, evaluate independent and jointly the impact of jitter and quantization to total uncertainty. Maximum jitter was fixed to *100* ns and *Nbits* was set to *16*. The ratio was set to low and *15457* samples were considered (M=625). Tables 2, 3 and 4 show results considering respectively jitter, quantization and both effects on the three methods, for amplitude and phase of the fundamental frequency.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Ej{ΔA1} (µV)** | **σj (ΔA1) (µV)** | **Ej{ΔΦ1} (µrad)** | **σj (ΔΦ1) (µrad)** |
| **DFT** | 1.06E-04 | 5.50E-02 | 6.69E-04 | 9.55E-02 |
| **MPSF** | -1.09E-04 | 5.49E-02 | -6.22E-04 | 1.89E-01 |
| **MANNFM** | -1.11E-04 | 5.49E-02 | -6.21E-04 | 1.89E-01 |

**Table 2.** Mean (*E*) and standard deviation (*σ*) of fundamental frequency deviations to nominal amplitude (*ΔA1*) and phase (*ΔΦ1*) for three methods - only jitter considered

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Eq{ΔA1} (µV)** | **σq (ΔA1) (µV)** | **Eq{ΔΦ1} (µrad)** | **σq (ΔΦ1) (µrad)** |
| **DFT** | 6.25E-04 | 1.12E-01 | 1.26E-04 | 1.12E-01 |
| **MPSF** | 1.16E-03 | 1.12E-01 | -1.28E-03 | 2.41E-01 |
| **MANNFM** | 1.15E-03 | 1.12E-01 | -1.28E-03 | 2.41E-01 |

**Table 3.** Mean (*E*) and standard deviation (*σ*) of fundamental frequency deviations to nominal amplitude (*ΔA1*) and phase (*ΔΦ1*) for three methods - only quantization considered

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Ejq{ΔA1} (µV)** | **σjq (ΔA1) (µV)** | **Ejq{ΔΦ1} (µrad)** | **σjq (ΔΦ1) (µrad)** |
| **DFT** | 7.31E-04 | 1.25E-01 | 7.94E-04 | 1.53E-01 |
| **MPSF** | 1.05E-03 | 1.33E-01 | -1.91E-03 | 3.06E-01 |
| **MANNFM** | 1.05E-03 | 1.33E-01 | -1.91E-03 | 3.06E-01 |

**Table 4.** **.** Mean (*E*) and standard deviation (*σ*) of fundamental frequency deviations to nominal amplitude (*ΔA1*) and phase (*ΔΦ1*) for three methods - jitter and quantization considered

Several conclusions can be reached with these results. Firstly, the mean deviations to nominal for amplitude and phase are around zero, always lower to 2·10-9 volts or 2·10-9 radians. These values have been repeated with the rest of *R*, *N*, jitter and *Nbits* values. So, from now, we will present only standard deviation values will be presented. Additionally, for the three methods, the phase standard deviation is lightly higher than amplitude standard deviation.

Concerning the fitting methods, results show that both provide almost identical results, which is logical, due both methods solve the same optimization problem in different ways. And comparing DFT and fitting methods, both provides very similar results in amplitude, with practically identical standard deviation, but higher standard deviations (in a factor around *2*) is obtained in phase by fitting methods.

Relating to jitter and quantization impact in the three methods results, for the values selected, although both affect to the total standard deviation, quantization has more impact than jitter. On the other hand, results show that their jointly impact is equivalent to the quadratic sum of the independent impacts. That can be explained because although quantization is an uncertainty over the amplitude value and jitter is an uncertainty over the time instant of sampling, jitter is in the last part a noise over the amplitude [1], so both contributions apply, at the end, to the same variable.

### Impact of *R* and *N* in results

In order to evaluate *R* and *N* impact on the methods results, for each combination of jitter and number of bit values, the 9 points from Table 1 were simulated. Tables 5 and 6 show standard deviation results obtained for amplitude and phase of the fundamental frequency in the case of jitter value of *100* ns and *16* bits of quantization. In order to unify data as much as possible, both tables simultaneously present values obtained considering only jitter (*j*), only quantization (*q*) and both (*jq*).

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | N | | | | | | | | |
|  |  | 1311 | | | 7073 | | | 15457 | | |
|  |  | R | | | R | | | R | | |
|  |  | 24 | 123 | 262 | 24 | 123 | 262 | 24 | 123 | 262 |
| DFT | q | 4.18E-01 | 4.18E-01 | 4.18E-01 | 1.86E-01 | 1.86E-01 | 1.86E-01 | 1.12E-01 | 1.12E-01 | 1.12E-01 |
| j | 1.88E-01 | 1.87E-01 | 1.88E-01 | 8.08E-02 | 8.08E-02 | 8.09E-02 | 5.47E-02 | 5.47E-02 | 5.47E-02 |
| jq | 4.60E-01 | 4.60E-01 | 4.59E-01 | 1.82E-01 | 1.82E-01 | 1.82E-01 | 1.25E-01 | 1.25E-01 | 1.24E-01 |
| MSFM | q | 4.14E-01 | 4.09E-01 | 4.15E-01 | 1.81E-01 | 1.77E-01 | 1.80E-01 | 1.20E-01 | 1.21E-01 | 1.21E-01 |
| j | 1.88E-01 | 1.88E-01 | 1.89E-01 | 8.08E-02 | 8.08E-02 | 8.08E-02 | 5.47E-02 | 5.46E-02 | 5.46E-02 |
| jq | 4.55E-01 | 4.53E-01 | 4.56E-01 | 1.97E-01 | 1.95E-01 | 1.96E-01 | 1.33E-01 | 1.32E-01 | 1.32E-01 |
| MANNFM | q | 4.14E-01 | 4.09E-01 | 4.15E-01 | 1.81E-01 | 1.77E-01 | 1.80E-01 | 1.20E-01 | 1.21E-01 | 1.21E-01 |
| j | 1.88E-01 | 1.88E-01 | 1.89E-01 | 8.08E-02 | 8.08E-02 | 8.08E-02 | 5.47E-02 | 5.46E-02 | 5.46E-02 |
| jq | 4.55E-01 | 4.53E-01 | 4.56E-01 | 1.97E-01 | 1.95E-01 | 1.96E-01 | 1.33E-01 | 1.32E-01 | 1.32E-01 |

**Table 5.** Standard deviation of fundamental frequency amplitude deviations to nominal values (µV) for three methods in Table 1 points, considering: only jitter (*j*), only quantization (*q*) and both (*jq*)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | N | | | | | | | | |
|  |  | 1311 | | | 7073 | | | 15457 | | |
|  |  | R | | | R | | | R | | |
|  |  | 24 | 123 | 262 | 24 | 123 | 262 | 24 | 123 | 262 |
| DFT | q | 4.14E-01 | 4.14E-01 | 4.14E-01 | 2.12E-01 | 2.12E-01 | 2.12E-01 | 1.21E-01 | 1.21E-01 | 1.21E-01 |
| j | 3.25E-01 | 3.25E-01 | 3.25E-01 | 1.40E-01 | 1.40E-01 | 1.40E-01 | 9.47E-02 | 9.47E-02 | 9.47E-02 |
| jq | 5.27E-01 | 5.26E-01 | 5.26E-01 | 2.26E-01 | 2.26E-01 | 2.27E-01 | 1.53E-01 | 1.53E-01 | 1.53E-01 |
| MSFM | q | 8.27E-01 | 8.24E-01 | 8.19E-01 | 3.54E-01 | 3.56E-01 | 3.54E-01 | 2.41E-01 | 2.40E-01 | 2.41E-01 |
| j | 6.50E-01 | 6.51E-01 | 6.55E-01 | 2.80E-01 | 2.80E-01 | 2.80E-01 | 1.89E-01 | 1.89E-01 | 1.89E-01 |
| jq | 1.05E+00 | 1.05E+00 | 1.06E+00 | 4.53E-01 | 4.53E-01 | 4.53E-01 | 3.06E-01 | 3.06E-01 | 3.06E-01 |
| MANNFM | q | 8.27E-01 | 8.24E-01 | 8.19E-01 | 3.54E-01 | 3.56E-01 | 3.54E-01 | 2.41E-01 | 2.40E-01 | 2.41E-01 |
| j | 6.50E-01 | 6.51E-01 | 6.55E-01 | 2.80E-01 | 2.80E-01 | 2.80E-01 | 1.89E-01 | 1.89E-01 | 1.89E-01 |
| jq | 1.05E+00 | 1.05E+00 | 1.06E+00 | 4.53E-01 | 4.53E-01 | 4.53E-01 | 3.06E-01 | 3.06E-01 | 3.06E-01 |

**Table 6.** Standard deviation of fundamental frequency phase deviations to nominals values (µrad) for three methods in Table 1 points, considering: only jitter (j), only quantization (q) and both (jq)

Firstly, tables 5 and 6 confirm in a more general way conclusions obtained in previous sections about methods comparisons. Moreover, from both tables it can be observed that for three methods and three cases *j*, *q* and *jq*, the *R* value does not affect the standard deviations of amplitude and phase, while the *N* value impacts clearly in the results, so that lower *N* values produces higher standard deviations. The same behaviour can be observed for the rest of jitter and *Nbits* values combinations.

### Impact of jitter and number of bits on results

In order to evaluate jitter and resolution impact in the three methods, each combination of *Maxjit* and *Nbits* values defined in section 4.2 were considered for simulation. Regarding *R*, taking into account previous test results, a single value (the intermediate) was fixed. Finally, the three *N* (and *M*) values were additionally considered, in order to observe the *N* influence.

So, *18* simulation points were performed. In all of them, results obtained consolidate previous tests conclusions: the fitting methods provide identical results which, at the same time respect DFT results, are identical in amplitude and higher by a factor around *around 2* in phase sigma. For this reason and with the aim of synthetize, only MANNFM results will be presented in this section, but all conclusions obtained can be applied to DFT and MSFM.

Results for MANNFM are presented in Tables 7 and 8, which show standard deviations obtained for amplitude and phase of the fundamental frequency at any simulation point. In order to unify data as much as possible, both tables simultaneously present values obtained considering only jitter (*j*), only quantization (*q*) and both (*jq*).

First of all, again previous conclusions are confirmed in more general test: as in previous section, higher *N* values results in lower standard deviations for both only jitter and only quantization.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | N | | | | | | | | |
|  |  | 1311 | | | 7073 | | | 15457 | | |
|  |  | Nbits | | | Nbits | | | Nbits | | |
|  |  | 16 | 18 | 21 | 16 | 18 | 21 | 16 | 18 | 21 |
| Maxjit 3 ps | j | 5.68E-06 | 5.68E-06 | 5.68E-06 | 2.42E-06 | 2.42E-06 | 2.42E-06 | 1.64E-06 | 1.64E-06 | 1.64E-06 |
| q | 3.94E-01 | 1.03E-01 | 1.27E-02 | 1.76E-01 | 4.49E-02 | 5.54E-03 | 1.18E-01 | 2.96E-02 | 3.84E-03 |
| jq | 3.94E-01 | 1.03E-01 | 1.27E-02 | 1.76E-01 | 4.49E-02 | 5.55E-03 | 1.18E-01 | 2.96E-02 | 3.84E-03 |
| Maxjit 100 ns | j | 1.89E-01 | 1.89E-01 | 1.89E-01 | 8.05E-02 | 8.05E-02 | 8.05E-02 | 5.48E-02 | 5.48E-02 | 5.48E-02 |
| q | 3.94E-01 | 1.03E-01 | 1.27E-02 | 1.76E-01 | 4.49E-02 | 5.54E-03 | 1.18E-01 | 2.96E-02 | 3.84E-03 |
| Jq | 4.51E-01 | 2.16E-01 | 1.90E-01 | 1.95E-01 | 9.26E-02 | 8.07E-02 | 1.31E-01 | 6.29E-02 | 5.50E-02 |

**Table 7.** Standard deviation of fundamental frequency amplitude deviations to nominal values (µV) for *Maxjit*, *Nbits* and *N* values simulated, considering: only jitter (*j*), only quantization (*q*) and both (*jq*)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | N | | | | | | | | |
|  |  | 1311 | | | 7073 | | | 15457 | | |
|  |  | Nbits | | | Nbits | | | Nbits | | |
|  |  | 16 | 18 | 21 | 16 | 18 | 21 | 16 | 18 | 21 |
| Maxjit 3 ps | j | 1.97E-05 | 1.97E-05 | 1.97E-05 | 8.33E-06 | 8.33E-06 | 8.33E-06 | 5.68E-06 | 5.68E-06 | 5.68E-06 |
| q | 8.68E-01 | 2.04E-01 | 2.63E-02 | 3.53E-01 | 8.82E-02 | 1.10E-02 | 2.36E-01 | 6.05E-02 | 7.42E-03 |
| jq | 8.68E-01 | 2.03E-01 | 2.63E-02 | 3.53E-01 | 8.83E-02 | 1.10E-02 | 2.36E-01 | 6.04E-02 | 7.43E-03 |
| Maxjit 100 ns | j | 6.56E-01 | 6.56E-01 | 6.56E-01 | 2.78E-01 | 2.78E-01 | 2.78E-01 | 1.89E-01 | 1.89E-01 | 1.89E-01 |
| q | 8.68E-01 | 2.04E-01 | 2.63E-02 | 3.53E-01 | 8.82E-02 | 1.10E-02 | 2.36E-01 | 6.05E-02 | 7.42E-03 |
| jq | 1.06E+00 | 6.87E-01 | 6.56E-01 | 4.58E-01 | 2.90E-01 | 2.78E-01 | 3.04E-01 | 1.99E-01 | 1.89E-01 |

**Table 8.** Standard deviation of fundamental frequency phase deviations to nominal values (µrad) for *Maxjit*, *Nbits* and *N* values simulated, considering: only jitter (*j*), only quantization (*q*) and both (*jq*)

Regarding *Maxjit* effect on *j* results, for the lower *Maxjit* value, *3* ps, we can observe very low standard deviation values, with maxima (for lower *N*) of *5.7·10-12* V and *19.7·10-12* rad. On the other hand, for 100 ns, we can observe maxima standard deviations values of *0.2·10-6* V and *0.6·10-6* rad.

Regarding the quantization effect on points selected, we can observe in Figure 4 that *N* and *Nbits* variations have comparable effects on the standard deviations values obtained. This can be useful to compensate the effects of one parameter with the other. For example, as we can observe in Figure 4, the standard deviation obtained for *18* bits and *1311* samples can be nearly obtained with 16 bits if *15457* samples are considered.

Finally, regarding the combined impact of jitter and quantization, it can be observed from tables 7 and 8 that in the low jitter point, impact of jitter is no significant in front of quantization impact, so that the combined *(jq)* standard deviations are equal to those of the only quantization considered case *(q)*.

In the case of *Maxjitt* equal100 ns more similar contributions are obtained, so that both effects contribute to the combined impact. Even so, with the lower *N* and *Nbits* values the main contribution is due to quantization although jitter also contributes. So, if *Nbits* raise, at lower *N*, the quantization contributions decreases; till with 21 bits, the main contribution is due to jitter. When *N* value is increased to it next value, the same behavior is observed, changing the main contribution depending on the *Nbits* value.

*N*

**Fig. 4.** Standard deviation of fundamental frequency amplitude deviations to nominal values (µV) obtained for *Nbits* and *N* values - only quantization considered

1. Conclusions and Future Work

Analysis performed has provided practical values of MANNFM standard deviation (amplitude and phase of the fundamental) for its use in practical measurements in laboratory. In the worst case, obtained results show uncertainties - for 95.5% of confidence level (2σ) - lower than *1.7* µV and *2.12* µrad for amplitude and phase respectively. From the methods comparison performed, it has been found that fitting methods, MANNFM and MSFM, provides identical results. Respect DFT, fitting methods are very similar in amplitude but their standard deviation for the phase of the fundamental is higher than that of DFT by a factor around *2*. Besides that, behavior of fitting methods regarding uncertainty contributions (jitter sampling and quantization noise) and sampling parameters (*R* and *N*) follow the same tendencies than DFT.

Further work is also necessary in order to study uncertainties for fundamental and harmonics components when signals with harmonic content are sampled. Other frequencies have to be studied also, in order to identify jitter effect when higher fundamental frequencies are necessary.

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1. Although there is a coincidence between some terms used in GUM and this work nomenclatures, since there is no possibility of confusion, GUM nomenclature has been maintained in this section in order to simplify its reading. [↑](#footnote-ref-1)