MATHEMATICS IN THE WORK OF SPINOZA AND GUARINI

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Introduction

During the seventeenth century mathematics and the exact sciences brought about a scientific revolution, and seemed to be involved in all novel social developments of the time. To give just a few examples, Descartes (1569-1650) used mathematics as a model for his metaphysics, his main concern for many years. His greatest legacies, for the purposes and framing of this paper, are related with moving classical geometry within the reach of algebra, putting into connection Euclid's and Vitruvius's theories, and also with the mathematisation of nature. He was responsible for the geometrical representation of algebraic equations, with the invention of Cartesian coordinates, which had a great influence within the field of architecture, translating these theories to the building experience of that period. Newton (1643-1727) on the other end used mathematical principles to explain the philosophy of nature in his *Philosophiae Naturalis Principia Mathematica*. He considered geometric synthesis superior to algebra, as it was much more suitable for justification and explanation of theories, and made possible a deeper insight of the problems (Folina 2012).

These approaches are also applied to architecture, in relation with compositional and aesthetic issues. The Euclid's approach based on his studies on geometry lead to the first written definition of the golden ratio, found in *Elements of Geometry* (Book 6, Definition 2): "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser" (Fitzpatrick 2007:156). This approach was widely used during the middle age by building guilds to define the proportional elevation of cathedrals and castles through the application of inscribed squares rules (Rykwer 1982:71). On the other side we have Vitruvius' approach based on *commodulatio*, or the definition of a module, usually the half of a column's diameter, expressed as a number which made possible to calculate the dimensions of all the other architectural elements (Sbacchi 2001). It was used not only during the Roman Empire, but also during the Renaissance as a powerful compositional rule, above all by Brunelleschi in his Basilica of the Holy Spirit in Florence. They are really different approaches to construction:

> The difference between the two system is a significant one: according to the Vitruvian, multiplications and subdivisions of numbers regulated architectural shapes and dimensions, adopting Euclidean constructions, instead, architecture and its elements were made out of lines, by means of compass and straightedge (Sbacchi 2001:27).

If there is something in common between geometry and algebra, it is the infinite, which gain an important role in art and culture during the 17th century. The historicist view of this period by Wölfflin in fact explains baroque architecture, but also art, literature and music, as based on the premonition of the infinite, on the search for the eternity. The influences of the many religious orders consolidating at this time, and the religious symbolism are also key to understand the culture of this time and the development of the gigantism in religious and civil architecture. It is particularly interesting to observe how the development of mathematics influences the architecture of the Baroque period, with a permanent effort on the research related with how to represent an infinite space. This led to the use of complex geometrical forms being the most interesting result the church Sant'Ivo alla Sapienza built in 1642-1660 by Francesco Borromini. The use of more complex forms than in previous periods, like oval plans and domes or rib vaults is only possible if we consider the great developments made into this science (Duvernoy 2015; Compán et al. 2015; Fallacara et al. 2011). Stereotomy, consolidated as a science in the 17th century, and descriptive geometry are at the base of these innovations, as scientific and theoretical approximations are needed to build these elements, only possible to be obtained through mathematics. Architecture and mathematics for instance are strongly linked and justify one to the other: architecture is often based on (complex) mathematical rules, and at the same time can be seen as a visual representation of mathematical functions.

Mathematics in the work of Spinoza

The work of Spinoza, like other sixteen hundreds philosophers, looks for an alternative to humanistic anthropocentrism. Like the architecture of the period which looked at the representation of divine and the infinite through central and longitudinal spaces organized around a generating axis, philosophy looked at the way to combine the transcendent infinite of the universe with the limited earthly horizon (Gritella 2013). Spinoza mentions only sporadically the art of building in his writings, basically as examples to explain his thoughts (EIII P2S; EIV Pref.). It would be possible to say that building and architecture are not interesting him, also if construction is a power concept pervading all his work. His theorization of the infinite is of great relevance not only related with the infinite of God's attributes, but also as an independent mathematical concept. In fact mathematics had a great importance in Spinoza's works; the point is often made about his mathematical way of reasoning, as applied in his *Ethics*. Known are his relations with many mathematicians, like the statesman Johan de Witt, author of *Elementa curvarum linearum,* a book on analytic geometry, or Johan Hudde, mayor of Amsterdam between 1672 and 1703 and also a mathematician involved in the translation of Descartes's Geometry into Latin. Moreover one forth of the books owned by Spinoza and sold after his death delved with mathematics or other scientific issues (Sánchez and Masri 2011).

To look at the possible influences he had over the built environment of his time and on baroque architecture we have to look indirectly to his conception about mathematics, which will be compared with the theoretical and practical activity of another great personality of the time, the Italian theologian, philosopher, mathematician and architect Guarino Guarini. There are no evidences of any contact or epistolary exchanges between Spinoza and Guarini despite the many trips the latter made to France, Portugal and Spain. Indirect influences should be investigated, but at the moment are only hypothetical conjectures. This lack of evidences, together with the repulsion the work of Spinoza had on the Dutch society of his time, apart from a reduced circle of friends interested in his work (Nadler 2006), led to think about the approach to mathematics in the work of these authors.

Spinoza values the important role of mathematics in the same *Ethics*, despite he also uses other methods to present his work: "So they maintained it as certain that the Gods' judgements far surpass man's grasp. This alone would have caused the truth to be hidden from the human race for ever, if mathematics hadn't shown them another standard of truth." (E1, App.). According with Sánchez y Masri (2011), not only *The Ethics* is developed according with mathematical principles, but its same philosophy can be read in mathematical terms, applying the set theory developed later on by Cantor. Sánchez and Masri (2011) propose a new reading of *The Ethics* substituting words with symbols, showing how Spinoza introduced in his epoch new mathematical concepts. The three senses of infinite advocated by Spinoza are defined by Bussotti and Tapp (2009) as in which it is said of God, in which it is said of God's attributes, and in which it is said of the *modi*, sharing the common feature that "they are actualizations of a potentially infinite process". Dea (2011) makes an interesting similitude associating Spinoza's infinite with indefiniteness, following Newton, Leibniz and finally Wolfson understanding of the infinite as indeterminate. Comparing the idea of infinite with a grammatical one, We will see at the end of this paper as this definition, at a first glance irreconcilable with the mathematical approach used in architecture, can be reconduced to the experiences held during the same years by Guarini.

Guarino Guarini

Guarino Guarini (1624-1683), trained in philosophy, metaphysic and theology during his novitiate in Rome under the Theatine order, has been rediscovered only recently, starting with the International Conference *Guarino Guarini e l'Internazionalità del Barocco*, held in Torino in 1968. The years he spent in Rome, during an effervescent period for the baroque architecture, he eventually met Borromini, who possibly influenced his future experience as architect. During the following years he moved across Europe (most important were his stays in Messina and Paris) teaching philosophy and mathematics, having the opportunity to enter in contact with many researchers and improving his knowledge. He settled definitely in Turin in 1666, a period of maturity when he develop his most famous projects, under the protection of the Savoys' monarchy. From the early years of his ordination he is involved in architectural design and management, being also the reason for his nomadic experiences, as he was required to follow the works of new churches to be built around Europe for the order. Not all the designs he realised finally were built, but the drawings we conserve, together with the built examples are interesting as for the strict relationship with mathematics and the innovative architectural language applied. As he himself states in his writings, mathematics is his favourite discipline, which can be easily understood when related with his activity as an architect. The most important theoretical works are *Placita Philosophica* (1665) published in Paris, where he learned about French philosophers active at that time; the treatise on theoretical and applied geometry *Euclides Adauctus et Methdicus Mathematicaque Universalis* (1671), where he explains the works of Euclid and other mathematicians and includes his own outcomes related with complex solids to make accessible. In its preface he states “the value and usefulness that this kind of work can have to irradiate with mathematical light and make evident all things with a single luminous source”, and many other quotes refer directly to the dependence of the art of building on mathematics (Roero 2009:418). Similar to Spinoza, his most recognised work *Architettura Civile* (1737), a compendium devoted to geometrical representation and architecture, is published only after his death in 1683 by Bernardo Vittone. We can realise how his broader interests are not only related with teaching philosophy and with architectural practice, as he also published many works which at a whole can be understood with the attempt to a disciplinarian dissemination of knowledge (about logic, physics, astronomy, stereometry...), always with a strong foundation on geometry as a tool to explain the idea of universal complexity (Caliendo 2009).

Guarini develops in *Euclides Adauctus* and in *Architettura Civile* a relationship between Euclid's geometry and architecture, applying Projection geometry developed by Girard Desargues in Paris 1639 to the planar projection of spherical surfaces.~~ He reasons that actual infinite, the only one accepted by Spinoza, cannot be admitted (based on several external established sources)~~. It is obvious how based on his theological approach, the development of the idea of infinite is always related with the existence of God. His approach focuses on the idea of quantity, using the different entities he describes as indivisible, such as the point, the line and the surface, which he perceives much more related with the continuity of the real than algebra. It is possible to find a direct connection between his idea of infinite and geometry, and cannot surprise that the *Euclides Adauctus*, subdivided into 35 books, is a broad recompilation and explication of theoretical and applied geometry. The use of geometry is hence related with the concept of continuity introduced together with his theological believes to build the idea of absolute and infinite, a concept well described in the treatise (Roero, 2009; Paleari). The studies of McQuillan on Guarini's philosophy and mathematics show the knowledge and control he reached with the application of infinite to mathematics and to his architecture:

> Guarini's mathematics in the context of the culture of the Baroque will lead to a surer confirmation of his greatness and his intellectual superiority in that 'century of genius', and to place him alongside the great figures of European architecture not because he was like them, but because he understood what he meant to a degree that surpassed them all. (McQuillan 2009:347)

The approach to infinite in the architecture of Guarini

The architecture realised by Guarini has the value to have been developed at the same time of his various treatises, and it is in this way a confirmation of his theories and at the same time its application into practice. Basically he was moving around

Europe to respond to the needs to design and manage new buildings for the Theatine order, until he settled in Torino, where he started to receive civil commissions, being the Savoys the most relevant one. We know most of his projects for the drawings published after his death, as several of the buildings disappeared due to earthquakes (Messina, Lisbon), were not built or simply were substantially transformed during their construction. Others, like San Lorenzo and the Sacred Shred Chapel in Torino, are among the most interesting masterpieces of the late Baroque in Italy. Many authors find in his work a deep influence from the work of Borromini which he could have met in Rome during his studies, but as Wittkower (1999: 33) mentions, the aims of the architects were different: Borromini attempted to create homogeneous spaces, while Guarini worked with inconsistencies and surprising dissonances, creating always a doubt about what will appear after any other element. I will in part disagree with this affirmation defined as 'atomization' of spaces, as while it may be true when realizing a detailed analysis one by one of the subsequent elements juxtaposed by Guarini in his work, when looking at his buildings as a whole, in a spatial way, these dissonances and inconsistencies disappear. It can be perceived a sensation of infinite and continuous space.

We can see this idea in the longitudinal plan for St Mary Our Lady of Divine Providence in Lisbon, where a continuity of curved walls, originated from a succession of ovals either in the main nave as in the secondary ones where the main axis is rotated 90 degrees, define this peculiar plan. Despite the strong division of the inner space into modules, looking at the plate of the section, a great continuity, also if realised into several planes, can be appreciated. It is not clear if the church was built according with Guarini's design, due to its disappearance after the 1755 earthquake (Meek 1998: 13). The limited graphical documentation we have, published on *Architettura Civile*, makes it difficult to fully comprehend the final result of this design, and how Guarini conceived it. Similar solutions which could give an idea of this space, without any intention to relate the design process developed to define them, could be the Church of San Miguel in Madrid (1739-46) by Giacomo Bonavía or the Church of San Marco in Madrid 1749-53) by Ventura Rodríguez.

In 1668 Guarini designed a centralized church dedicated to San Lorenzo, on the place of a previous chapel with important meanings for the Royal family, remembering the victory in the Battle of Saint Quentin more than one century earlier, key point for the return of the region of Savoy to this dynasty. The plan of the church, an octagon with convex edges inscribed in a square, develops all its complex geometry on its central vertical axis. Four different levels can be recognised, with the alternation of convex and concave plains which create complex and rich geometrical plains, according with the idea of *pulsing organisms*. The ground level, where the octagon is recognizable, presents eight serlianas on convex surfaces. On top of this level, alternate convex serlianas and concave pendentive continue the previous composition. As the serlianas are framed by round arches, this level concludes in a circular shape. The third level is possibly the more original one, a dome defined by eight ribs which create an octagon star, remembering Islamic geometrical shapes. Between the adjacent footings of the ribs new pendentive terminate in convex geometries, which circumscribes an inner octagon. The last level is composed by an octagonal drum with windows separated by eight columns. On top of these, eight crossed arches creates the octagonal base for the last dome, which crown the telescopic development. Each of the level has a set of windows which create a space filled of light, and also if ribs have been used before – Borromini in Palazzo di Propaganda Fide and in the Oratory dei Filippini in Rome – the use given by Guarini is quiet different, being the starting point for a much more complex configuration. Definitely the octagon created by the ribs is a window opening on a further extension of space developed with the *ars combinatoria*, which will characterize this architect.

Better known is the Chapel of the Sacred Shred in Turin, also if actually under restoration after a fire in 1997 compromised its structure and interiors. Like in San Lorenzo and in the disappeared Church of Somaschi Fathers in Messina, Guarini adopted the specific solution of the telescopic dome. The original project started by Castellamonte in 1657 was replaced by Guarini in 1666. Eventually the spherical dome on the circular chapel was substituted by a much more intellectual and experimental solution, based on several perfect shapes and on the multiples of number three (representing the Trinity). Above the cylindrical space of the chapel three arches alternated by pediments, configure a first module, followed by a "traditional" drum composed of six arched windows. Six arches follow this level, laying, surprisingly on the centre of these windows, which are followed by other five groups of six arches, each of them smaller and rotated 30 degrees, for a total of 36 arches creating a dome, known as the basket. These arches also acts as windows, creating a great illumination of the inner chapel, strengthened by the materials used, which pass from the black marble to a lighter grey one, moving from the mourning for the death of Christ (black) at the lower level to the deliverance coming from the sky (light grey) at the higher level, a common solution during Mannerism and Baroque. The geometry of the hexagon which circumscribe the edges of the arches (which is possible to inscribe in a circle, and can be divided into six equilateral triangles) and the triangle are present, among many other religious and mysterious symbols. The last two elements are the eyes, a twelve-pointed star with more openings, and the pinnacle or lantern, which also is composed of three sub-levels. As it can be seen from the images, it is a very complex system, also from the structural point of view, but when seen from the ground, it is perceived with a sensation of continuity and infinite, despite the real compression of some elements like the basket to create an illusion of deep. ~~According with Norbeg-Schulz () the chapel "is one of the most mysterious and fascinating spaces ever created".~~

Why the infinite?

But why focusing on the infinite? If geometry is the study of the mathematical properties of the space, architecture could be seen as the semiotic of spatial forms. Geometry and architecture converge into topology, understood as the research of the continuity of the forms, which is the point where form meet the infinite, as continuity leads to the *real infinite***,** and not only to the *potential one*. The analysis of the architecture built by Guarini, also substantiated by his theoretical work, shows the attempt he realised to introduce infinite into architecture, using innovative solutions which will influence later baroque in Austria and the South of Germany. If previous architecture adopted hemispherical or pointed domes as the representation of the sky and lanterns as a way to infill this space with light, the specific solution given by Guarini can be seen as a further development of this concept, looking at the infinite as a connection with God. Likewise, the composition of different volumes can be read as the attempt to run away from the form to reach the indetermination, and as a result the infinite. This way the direct relation with his theoretical works where geometry had a great weight take on importance. The analysis done by Vasileios (2009) on the church of San Lorenzo, where he read the complexity of its elements through the Deleuzian theory of the fold, tend to confirm this hypothesis of an infinite work in process. The fold tend to create a connexion between the different and contradictory or dissonant elements to create a single continuity.

Guarini is also the best representative of the increasing influence of mathematics applied to architecture, a trend which last until today. The architectures presented in this paper can be related with the work on eternity and infinitesimal infinite realised by Spinoza. Also if there are no evidences of direct contacts between the two authors, as also their seminal works were published after their respective death, we believe that given sectors of the society of the 17th century was permeated by these themes, leading to the results we know. Also if infinite theorized by Spinoza is not directly related with mathematics, understanding it as indefinite, its approximation to the geometrical space open new opportunities to understanding in the architectural field, being the telescopic domes built by Guarini meaningful examples.

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