The Fluctuating Two-Ray Fading Model: Exact and Approximate Statistical Characterization

Juan M. Romero-Jerez, F. Javier Lopez-Martinez, José F. Paris and Andrea J. Goldsmith

***Abstract*— We introduce the Fluctuating Two-Ray (FTR) fading model, a new statistical channel model that consists of two fluctuating specular components with random phases plus a diffuse component. The PDF and MGF are expressed in closed-form, having a functional form similar to other state-of-the-art fading models. We also provide an approximate closed-form expressions for the PDF, which allow for a simple evaluation of these statistics to an arbitrary level of precision. We show that the FTR fading model provides a much better fit than Rician fading for recent small-scale fading measurements in 28 GHz outdoor millimeter-wave channels.**

I. Introduction

Very recently [1], the small-scale fading statistics obtained from a 28 GHz outdoor measurement campaign showed that Rician fading was more suited than Rayleigh even in NLOS environments. However, a deeper look into the results of [1] indicates that conventional fading models in the literature fall short in accurately modeling the random fluctuations suffered by the received signal. We here propose a new model to capture this behavior: the Fluctuating Two-Ray (FTR) fading model, as a natural generalization of the TWDP fading model by allowing the constant-amplitude specular waves associated to LOS propagation to randomly fluctuate. Remarkably, this larger flexibility does not come at the price of an increased mathematical complexity, but instead facilitates a simpler statistical characterization than the TWDP model.

II. Statistical characterization of the FTR model

Let us consider that the complex baseband received signal can be written as

$$V_r = \sqrt{\zeta} V_1 \exp\left(j\phi_1\right) + \sqrt{\zeta} V_2 \exp\left(j\phi_2\right) + X + jY. \quad (1)$$

where $\phi_1, \phi_2 \sim \mathcal{U}[0, 2\pi)$, $X, Y \sim \mathcal{N}(0, \sigma^2)$ and ζ is a unit-mean Gamma distributed random variable with PDF

$$f_\zeta\left(u\right) = \frac{m^m u^{m-1}}{\Gamma\left(m\right)} e^{-mu}. \quad (2)$$

J. M. Romero-Jerez is with Departmento de Tecnología Electrónica, Universidad de Malaga, Malaga 29071, Spain. Contact e-mail: romero@dte.uma.es. F. J. Lopez-Martinez and J. F. Paris are with Departmento de Ingeniería de Comunicaciones, Universidad de Malaga, Malaga 29071, Spain. A. Goldsmith is with the Wireless Systems Lab, Department of Electrical Engineering, Stanford University, CA, USA.

This work has been funded by the Universidad de Malaga - Campus de Excelencia Internacional Andalucía Tech, the Consejería de Economía, Innovación, Ciencia y Empleo of the Junta de Andalucía, the Spanish Government and the European Fund for Regional Development FEDER (projects P2011-TIC-7109, P2011-TIC-8238, TEC2014-57901-R, TEC2013-42711-R and TEC2013-44442-P

This model will be subsequently denoted as the Fluctuating Two-Ray (FTR) and is conveniently expressed in terms of the parameters K and Δ, defined as

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \qquad \Delta = \frac{2V_1 V_2}{V_1^2 + V_2^2}. \quad (3)$$

Lemma 1: Let us consider the FTR fading model as described in (1). Then, the MGF of the received SNR γ (or, equivalently, the power envelope) will be given by

$$\begin{aligned} M_\gamma\left(s\right) = {} & \frac{m^m\left(1+K\right)\left(1+K-\bar{\gamma}s\right)^{m-1}}{\left(\sqrt{\mathcal{R}\left(m,k,\Delta;s\right)}\right)^m} \\ & \times P_{m-1}\left(\frac{m\left(1+K\right)-\left(m+K\right)\bar{\gamma}s}{\sqrt{\mathcal{R}\left(m,k,\Delta;s\right)}}\right), \end{aligned} \quad (4)$$

where $\mathcal{R}\left(m,k,\Delta;s\right)$ is a polynomial in s defined as

$$\begin{aligned} R\left(m,k,\Delta;s\right) = {} & \left[\left(m+K\right)^2 - \Delta^2 K^2\right]\bar{\gamma}^2 s^2 \\ & - 2m\left(1+K\right)\left(m+K\right)\bar{\gamma}s + m^2\left(1+K\right)^2, \end{aligned} \quad (5)$$

$\bar{\gamma}$ is the average SNR and $P_\mu(\cdot)$ is the Legendre function of the first kind of degree μ.

Proof: See [2] ■

The FTR fading model introduced here is well-suited to recreate the propagation conditions in a wide variety of wireless scenarios, ranging from very favorable ones to worse-than Rayleigh fading. It also includes many important well-known statistical fading models as particular cases, i.e., TWDP, Rician shadowed, Rician, Rayleigh. one-sided Gaussian, Nakagami-*m* and Nakagami-*q* (Hoyt).

Lemma 2: When $m \in \mathbb{Z}^+$, the PDF of the SNR γ in a FTR fading channel can be expressed in terms of the confluent hypergeometric function $\Phi_2(\cdot)$ defined in [3, p. 34, (8)], as given in (7).

Proof: See [2] ■

Lemma 3: When $m \in \mathbb{Z}^+$, the PDF of the SNR γ in a FTR fading channel can be approximated by a finite sum of elementary functions, as given, in (8), where $M > \lceil K\Delta \rceil$, $\beta = \frac{K+1}{\bar{\gamma}}$ and the coefficients α_i and δ_i are defined as

$$\alpha_i = \frac{2(-1)^i}{(2M-1)(2M-i)!(i-1)!}\int_0^{2M-1}\prod_{\substack{k=1\\k=i}}^{2M}\left(u-k+i\right)du,$$

$$\delta_i = \Delta\cos\left(\frac{(i-i)\pi}{2M-1}\right). \quad (6)$$

Proof: See [2] ■

The PDF of the received signal envelope r can be easily derived by a simple change of variables. Specifically, $f_r(r) =$

$$f_\gamma(x) = \frac{1}{2^{m-1}}\frac{1+K}{\bar\gamma}\left(\frac{m}{\sqrt{(m+K)^2-K^2\Delta^2}}\right)^m \sum_{q=0}^{\lfloor (m-1)/2\rfloor}(-1)^q C_q^{m-1}\left(\frac{m+K}{\sqrt{(m+K)^2-K^2\Delta^2}}\right)^{m-1-2q} \times \Phi_2^{(4)}\Bigg(1+2q-m, m-q-\frac{1}{2}, m-q-\frac{1}{2}, 1-m; 1; -\frac{m(1+K)}{(m+K)\bar\gamma}x, -\frac{m(1+K)}{(m+K(1+\Delta))\bar\gamma}x, -\frac{m(1+K)}{(m+K(1-\Delta))\bar\gamma}x, -\frac{1+K}{\bar\gamma}x\Bigg). \tag{7}$$

$$\widehat{f}_\gamma(x) \approx \sum_{i=1}^{M}\frac{\alpha_i}{2}\left\{\mathcal{G}_m(x;\beta,K(1-\delta_i)) + \mathcal{G}_m(x;\beta,K(1+\delta_i))\right\}, \tag{8}$$

$$\mathcal{G}_m(x;\beta,K) = \left(\frac{m}{K+m}\right)^m \beta e^{-\beta x}\sum_{n=0}^{m-1}\binom{m-1}{n}\left(\frac{K\beta x}{K+m}\right)^n\frac{1}{n!}; \tag{9}$$

$2rf_\gamma(r^2)$ and replacing $\bar\gamma$ by Ω, where $\Omega = E\{r^2\}$. In Fig. 1 we show the effect of the FTR fading model parameters K, Δ and m on the shape of the signal envelope PDF.

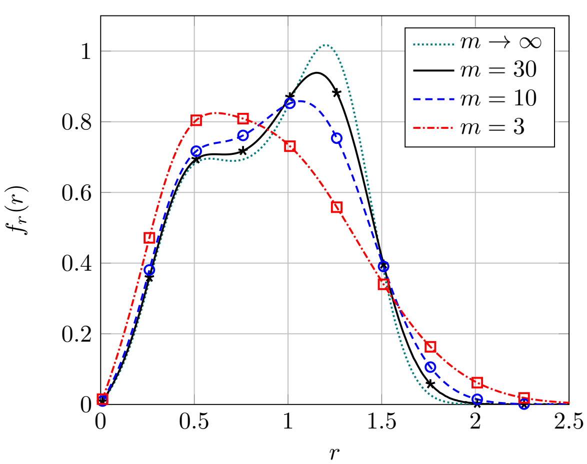

Fig. 1. FTR signal envelope for different values of m, with $K = 15$, $\Delta = 0.9$ and $\Omega = 1$. Solid lines: exact PDF. Markers: approximate PDF.

III. Empirical validation

We use the empirical results presented in [1] to validate the FTR fading model in the context of small-scale fading modeling of mmWave outdoor communications in the 28 GHz band. A modified version of the Kolmogorov-Smirnov (KS) statistic has been used to define the error factor ϵ used to quantify the goodness of fit between the empirical and theoretical CDFs, denoted by $\hat{F}_r(\cdot)$ and $F_r(\cdot)$ respectively, i.e,

$$\epsilon \triangleq \max_x |\log_{10}\hat{F}_r(x) - \log_{10}F_r(x)|. \tag{10}$$

In Fig. 2 we compare the set of measurements corresponding to the NLOS cross-polarized scenarios described in [1, Fig. 6]. For this set of measurements, the empirical CDFs lie within the theoretical CDFs corresponding to a Rician distribution with values of K ranging from 2 to 7 (i.e. 3 to 8 dB). According to the KS statistic, the values of K that provide the

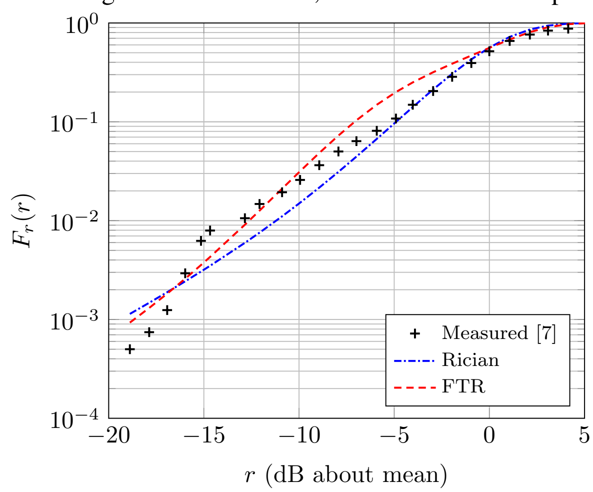

Fig. 2. Empirical vs theoretical CDFs of the received signal amplitude for NLOS scenario. Parameter values are $K_{\text{Rice}} = 4.78$ and $K_{\text{FTR}} = 32.7$, $\Delta = 0.8331$, $m = 10$. Measured data obtained from [1, Fig. 6, NLOS].

best fit to the Rician distribution is $K_{\text{NLOS}}^{\text{Rice}} = 4.78$. Such value of K yield an error factor value of $\epsilon_{\text{NLOS}}^{\text{Rice}} = 0.3571$. Now, using the proposed FTR fading model, we obtain the following set of parameters: $\text{FTR}_{\text{NLOS}} = (K = 32.7, \Delta = 0.8331, m = 10)$. The error factor value obtained by the FTR fit is $\epsilon_{\text{NLOS}}^{\text{FTR}} = 0.2681$. Thus, a remarkable improvement is attained when using the FTR fading model with respect to the Rician model.

References

[1] M. K. Samimi, G. R. MacCartney, S. Sun, and T. S. Rappaport, "28 GHz Millimeter-Wave Ultrawideband Small-Scale Fading Models in Wireless Channels," in *2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, May 2016.

[2] J. M. Romero-Jerez, F. J. Lopez-Martinez, J. F. Paris, and A. Goldsmith, "The Fluctuating Two-Ray Fading Model: Statistical Characterization and Performance Analysis," *arXiv preprint: arXiv:1611.05063v1*, 2016.

[3] P. W. K. H. M. Srivastava, *Multiple Gaussian Hypergeometric Series*. John Wiley & Sons, 1985.