Predicting the Electricity Demand Response via Data-driven Inverse Optimization

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Towards the decentralization of the electricity grid

↑ Complexity
↑ Information
↑ Robustness, resilience, reliability, security
Involving the small consumer
Outline

- Motivation
- Forecasting the price-responsive customers’ demand
- Defining the estimation problem
- Solving the estimation problem
- Case study: HVAC system of a pool of buildings
- Conclusions
A cluster of **price-responsive** consumers is considered

This cluster is expected to **consume** more at a **favorable price**

We describe the pool of price-responsive consumers as a **utility maximizer agent**

**Step-wise** marginal utility function
Consumers’ price-response model

\[ \begin{align*}
\text{maximize} \quad & \sum_{b=1}^{B} x_{b,t}(u_{b} - p_{t}) \\
\text{subject to} \quad & P \leq \sum_{b=1}^{B} x_{b,t} \leq \overline{P} \quad (\lambda_{t}, \overline{\lambda}_{t}) \\
& 0 \leq x_{b,t} \leq E_{b} \quad (\phi_{b,t}, \overline{\phi}_{b,t})
\end{align*} \]

It is a linear optimization problem (LOP).

Unknown variables:

- **Marginal utilities** \( u_{b} \)
- **Power bounds** \( \overline{P}, \underline{P} \)

We seek values of \( u_{b}, \overline{P}, \) and \( \underline{P} \) based on observations of \( x'_{b,t} \) and \( p_{t} \), given \( E_{b} \). We use the estimated utility maximizer problem to predict \( x_{t+1} \).
The estimation problem: Optimality condition

\[
\begin{align*}
\text{minimize} & \quad \sum_{t=1}^{T} \epsilon_t \\
\text{subject to} & \quad \overline{P} \overline{\lambda}_t - P \underline{\lambda}_t + \sum_{b=1}^{B} E_b \overline{\phi}_{b,t} - \epsilon_t = \sum_{b=1}^{B} x_{b,t}(u_b - p_t), \quad \forall t \\
& \quad \overline{\phi}_{b,t} - \phi_{b,t} + \overline{\lambda}_t - \underline{\lambda}_t = u_b - p_t, \quad \forall t \\
& \quad \overline{\phi}_{b,t}, \phi_{b,t}, \overline{\lambda}_t, \underline{\lambda}_t, \epsilon_t \geq 0, \quad \forall t
\end{align*}
\]

\[\Omega = \{ \epsilon_t, \overline{P}, P, u_b, \overline{\lambda}_t, \underline{\lambda}_t, \overline{\phi}_{b,t}, \phi_{b,t} \}\]

Inverse optimization (IOP) is used to determine the parameters of the model to make predictions of the load.
Leveraging auxiliary information

Model parameters $\overline{P}_t$, $P_t$ and $u_{b,t}$, might vary over time. We assume a number of time varying regressors $Z$ such that

\begin{align}
P_t &= \mu + \sum_{r=1}^{R} \alpha_r Z_{r,t} \\
\overline{P}_t &= \mu + \sum_{r=1}^{R} \overline{\alpha}_r Z_{r,t} \\
u_{b,t} &= \mu^u_b + \sum_{r=1}^{R} \alpha^u_r Z_{r,t}
\end{align}

Regressors relate to **time** and **weather**:
- Temperature of the air outside
- Solar irradiance
- Hour indicator
- Past price and load
Leveraging auxiliary information

The price-response model must make sense for any plausible value of the features, in particular,

- The minimum consumption limit must be lower than or equal to the maximum consumption limit
- The minimum consumption limit must be non-negative
Leveraging auxiliary information

For example,

\[ P_t = P + \sum_{r \in R} \alpha_r Z_{r,t} \leq \overline{P} + \sum_{r \in R} \overline{\alpha}_r Z_{r,t} = \overline{P}_t, \quad t \in T, \text{ for all } Z_{r,t} \]

Assume that \( Z_{r,t} \in [\underline{Z}_r, \overline{Z}_r] \), then

\[ P - \overline{P} + \max_{Z'_{r,t}} \left\{ \sum_{r \in R} (\alpha_r - \overline{\alpha}_r) Z'_{r,t} \right\} \leq 0, \quad t \in T. \]

which is equivalent to

\[ \overline{P} - P + \sum_{r \in R} (\overline{\phi}_{r,t} \overline{Z}_r - \underline{\phi}_{r,t} \underline{Z}_r) \leq 0 \quad t \in T \]

\[ \overline{\phi}_{r,t} - \underline{\phi}_{r,t} = \overline{\alpha}_r - \underline{\alpha}_r \quad r \in R, t \in T \]

\[ \underline{\phi}_{r,t}, \overline{\phi}_{r,t} \geq 0 \quad r \in R, t \in T. \]
Solving the estimation problem

- The estimation problem is **non-linear and non-convex**.

- We statistically **approximate its solution** by solving two **linear programming problems** instead.
  

- A **two-step data-driven estimation procedure** to achieve **optimality** and **feasibility** of $x'$ in a statistical sense.
Feasibility problem: Estimation of power bounds

\[
\begin{align*}
\text{Minimize} & \quad \sum_{t=1}^{T} \left( (1 - K) \left( \bar{\xi}_t^+ + \bar{\xi}_t^- \right) + K \left( \bar{\xi}_t^- + \bar{\xi}_t^+ \right) \right) \\
\text{subject to} & \quad \bar{P}_t - x_t' = \bar{\xi}_t^+ - \bar{\xi}_t^- \quad \forall t \\
& \quad x_t' - P_t = \bar{\xi}_t^+ - \bar{\xi}_t^- \quad \forall t \\
& \quad P_t \leq \bar{P}_t \quad \forall t \\
& \quad P_t = \bar{\mu} + \sum_{r=1}^{R} \alpha_r Z_{r,t} \quad \forall t \\
& \quad \bar{P}_t = \bar{\mu} + \sum_{r=1}^{R} \bar{\alpha}_r Z_{r,t} \quad \forall t \\
& \quad 0 \leq \bar{\xi}_t^+, \bar{\xi}_t^-, \xi_t^+, \xi_t^- \quad \forall t
\end{align*}
\]
Optimality problem: Estimating marginal utilities

Minimize \( \sum_{t=1}^{T} \epsilon_t \)

subject to

\[
\begin{align*}
\sum_{b=1}^{B} & \sum_{b=1}^{B} x'_{b,t} (u_{b,t} - p_t) \\
& \leq \sum_{b=1}^{B} \overline{\lambda}_{t} - \overline{\lambda}_{t} \lambda_{t} + \sum_{b=1}^{B} E_b \overline{\phi}_{b,t} - \epsilon_t = \\
& - \overline{\phi}_{b,t} + \overline{\phi}_{b,t} - \Delta_t + \overline{\lambda}_{t} = u_{b,t} - p_t \\
u_{b,t} &= \mu_{b}^{u} + \sum_{r} \alpha_{r}^{u} Z_{r,t} \\
\mu_{b}^{u} &\geq \mu_{b+1}^{u} \\
\mu_{1}^{u} &\geq 200 + \mu_{2}^{u} \\
0 &\leq \overline{\lambda}_{t}, \Delta_t, \overline{\phi}_{b,t}, \overline{\phi}_{b,t}
\end{align*}
\]
Solving the estimation problem

In the **bound estimation problem**, the **penalty parameter** $K$ is statistically tuned through **validation**:

- *(x, p, Z)* data set
- **Training set** → Used for parameter fitting for each possible value of $K$
- **Validation set** → Used to tune parameter $K$
- **Test set** → Used to assess forecasting performance

We choose $K$ so that the **out-of-simple prediction error** is minimized

**K** as indicator of the **price-responsiveness of the load**:

- $K = 0$ → **Narrow** interval → **Small variability** of the load explained by the price.
- $K = 1$ → **Wide** interval → **High variability** of the load explained by the price.
Case study (one-hour ahead prediction)

We simulate the price-response behavior of a pool of 100 buildings equipped with heat pumps (assuming economic MPC is in place).

Two classes of buildings are considered, depending on the comfort bands of the indoor temperature.
Case study

We conduct a benchmark of the methodology against simple persistence forecasting and autoregressive moving average with exogenous inputs.

- **Simple persistence model**: The forecast load at time $t$ is set to be equal to the observed load at $t - 1$.

- **ARMAX**: The aggregate load $x$ is a linear combination of the past values of the load, past errors and regressors.

\[
x_t = \mu + \epsilon_t + \sum_{p=1}^{P} \varphi_p x_{t-p} + \sum_{r=1}^{R} \gamma_r Z_{t-r} + \sum_{q=1}^{Q} \theta_q \epsilon_{t-q}
\]

Forecasting performance is evaluated according to MAE and

\[
NRMSE = \frac{1}{x_{\text{max}} - x_{\text{min}}} \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left( \sum_{b=1}^{B} \hat{x}_{b,t} - x_t' \right)^2}
\]

\[
MASE = \frac{\sum_{t=1}^{T} \left| \sum_{b=1}^{B} \hat{x}_{b,t} - x_t' \right|}{\frac{T}{T-1} \sum_{t=2}^{T} \left| x_t' - x_{t-1}' \right|}
\]
For an inflexible pool of loads, \( \text{InvFor} \approx \text{ARMAX} \). When the load aggregation is sensitive to the price, however, \( \text{InvFor} \) substantially outperforms \( \text{ARMAX} \).
Conclusions

What we have done:

• A new method to forecast price-responsive electricity consumption one step ahead.

• A two-step algorithm to statistically approximate the exact inverse-optimization solution.

• A validation scheme to minimize the out-of-sample prediction error.

• A methodology evaluation on a data set corresponding to a cluster of price-responsive buildings equipped with a heat pump.

The non-linearity between price and aggregate load is well described by our methodology.
Future Work

- Dealing with corrupted measurements.
- Examining more flexible functional forms between model parameters and regressors.
- Investigating statistically consistent set-valued functions (feasibility set as a function of regressors)
- Testing the methodology on other data sets.
Any questions?

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Full paper  
Short-term forecasting of price-responsive loads using inverse optimization 
is available online at IEEExplore  