Correlations in the magnitude of heartbeat increments as a measure of nonlinearity

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The talk

- We propose a measure nonlinearity for time series based on the analysis of the autocorrelation of the magnitude of the series.

- We apply it to series of interbeat intervals (RR-intervals) recorded during rest and exercise.
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Lack of nonlinearity $\Rightarrow$ PROBLEMS


Nonlinear time series

- Generated by nonlinear dynamical equations
- There exist correlations beyond the autocorrelation function (i.e. beyond linear correlations)

\[ C_x(\ell) = \frac{\langle x_i \cdot x_{i+\ell} \rangle - \langle x_i \rangle \langle x_{i+\ell} \rangle}{\sigma_x^2} \]

Multifractality
- Non-random Fourier phases (Schreiber & Schmitz, 2000)
- Correlations in the magnitude series

Nonlinear time series

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Correlations in the magnitude

- Given a time series \( \{y_i\}, \ i = 1, \ldots, N \) its magnitude series (also called volatility) is given by:
  \[
  |x_i| = |y_{i+1} - y_i|
  \]

- Correlations in \( |x_i| \) → Related to nonlinearity

- The decomposition into magnitude and sign has **Physiological meaning** (e.g. heartbeat fluctuations)

- Such correlations are quantified using DFA (Detrended Fluctuation Analysis)

Detrended Fluctuation Analysis

- Indirect measure of correlations (actually it measures fluctuations)
- Smooths out the noise in the autocorrelation function.

Is it always good?

- Only when autocorrelation function is a power law the results can be properly interpreted.
- Even having power laws there are problems with correlations in the magnitude (Carpena et al. 2017)

We propose here a direct study of the autocorrelation function of the magnitude

P. Carpena et al.: Spurious Results of Fluctuation Analysis Techniques in Magnitude and Sign Correlations. Entropy 19(6), 261 (2017)
Model for linearity $\Rightarrow$ **Linear Gaussian Noise**

- Let $\{x_i\}$ be a series of $\mathcal{N}(0,1)$ random variables with only linear correlations
Model for linearity ⇒ Linear Gaussian Noise

- Let \( \{x_i\} \) be a series of \( \mathcal{N}(0, 1) \) random variables with only linear correlations.

- If we denote by \( C_x(\ell) \) its autocorrelation function at distance \( \ell \), the autocorrelation function of its magnitudes, \( C_{|x|}(\ell) \), is given by:

\[
C_{|x|} = \frac{2 \left[ C_x \arcsin C_x - 1 + \sqrt{1 - C_x^2} \right]}{\pi - 2}
\]

- \( C_{|x|}(\ell) \geq 0 \) y \( C_{|x|}(\ell) = 0 \) ⇔ \( C_x(\ell) = 0 \)

- For small values of \( C_x \), we have: \( C_{|x|} = \frac{1}{\pi - 2} C_x^2 + \mathcal{O}(C_x^4) \)

**Examples of $C_{|x|}$ vs. $C_x$ for Gaussian Noises**

- The magnitude of a linear noise can be correlated

  $C_{|x|} \neq 0 \implies$ Nonlinearity
Examples of $C_{|x|}$ vs. $C_x$ for Gaussian Noises

- The magnitude of a linear noise can be correlated

\[ C_{|x|} \neq 0 \implies \text{Nonlinearity} \]

- The deviation from the theoretical curve can be a measure of nonlinearity
Heartrate during exercise

Heartrate increases and heartrate variability is reduced. The power spectrum is reduced, especially at low frequencies (respiration rate dominates). Sample entropy is reduced, and short-range correlations are reduced. It is not clear if the multifractal spectrum disappears. In general, complexity is reduced.
Heartrate during exercise

- Heartrate increases and heartrate variability is reduced
- Power spectrum is reduced, specially at low frequencies (respiration rate dominates)
- *Sample entropy* is reduced
- Short-range correlations are reduced (Not clear)
- Multifractal spectrum disappears (?)
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In general: Complexity is reduced
¿What about nonlinearity?

- More lineal ⇒ less complex

- Measure of nonlinearity: deviation from linear Gaussian expectation

\[
\Delta = \sum_{\ell=1}^{\ell_{\text{max}}} \delta C(\ell)^2
\]

where:

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\delta C(\ell) = C_{|x|}(\ell) - C_{|x|,\text{linear}}[C_x(\ell)]
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There’s no assumption of scaling or fractality in the autocorrelation function
Rest vs. exercise for football players

- We choose $\ell_{\text{max}} = 10$ beats
- Prior to the analysis data is converted into Gaussian
- For all subjects $\Delta$ is greater for rest, this is also true for group average.
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Prior to the analysis data is converted into Gaussian

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Higher nonlinearity during REST for professional football players (statistically significant $p = 0.047$)
Permanent effects of exercise on HR nonlinearity (cardio vs. strength training)

Higher nonlinearity during rest for football players (statistically significant $p = 7.6 \times 10^{-4}$)

- Soccer players
  - 22 males 23.0 ± 4.1 y/o
- Bodybuilders
  - 31 males 28.0 ± 6.1 y/o
Permanent effects of exercise on HR nonlinearity (cardio vs. strength training)

- Higher nonlinearity during rest for football players (statistically significant $p = 7.6 \times 10^{-4}$)
- Is aerobic training better for the heart?
Thank you for your attention