A formal model for explicit knowledge as awareness *of* plus awareness *that*

Claudia Fernández-Fernández[1], Fernando R. Velázquez-Quesada[2]

Model-based Reasoning in Science and Technology
24th - 26th October 2018
University of Seville (Spain)

[1] Universidad de Málaga (Spain), cffernandez@uma.es
[2] ILLC, Universiteit van Amsterdam (Netherlands), F.R.VelazquezQuesada@uva.nl

Explicit knowledge as awareness *of* + awareness *that*

In a nutshell

- ***Context:*** logical omniscience vs agents with limited abilities.

In a nutshell

- ***Context:*** logical omniscience vs agents with limited abilities.
- ***Purpose:*** reconsider what constitutes ***explicit knowledge***.

In a nutshell

- ***Context:*** logical omniscience vs agents with limited abilities.
- ***Purpose:*** reconsider what constitutes ***explicit knowledge***.
- ***Here:*** a formal model capturing the theoretical ideas.

Types of knowledge

- Different types of knowledge:

Types of knowledge

- Different types of knowledge:
  - ***explicit*** and ***implicit*** w.r.t. ***deduction*** (e.g., Konolige 1984, Levesque 1984);

Types of knowledge

- Different types of knowledge:
  - ***explicit*** and ***implicit*** w.r.t. ***deduction*** (e.g., Konolige 1984, Levesque 1984);
  - ***explicit*** and ***implicit*** w.r.t. ***awareness*** (Fagin and Halpern 1988);

Types of knowledge

- Different types of knowledge:
  - ***explicit*** and ***implicit*** w.r.t. ***deduction*** (e.g., Konolige 1984, Levesque 1984);
  - ***explicit*** and ***implicit*** w.r.t. ***awareness*** (Fagin and Halpern 1988);
- Note.
  - ***Explicit knowledge:*** what the agent ***actually has***.
  - ***'Implicit' knowledge:*** what she can ***reach via some given action***.

The ingredients

- ***Awareness-of*** and ***awareness-that***.

The ingredients

- ***Awareness-of*** and ***awareness-that***.
  - Fagin and Halpern (1988): awareness has ***different interpretations***.

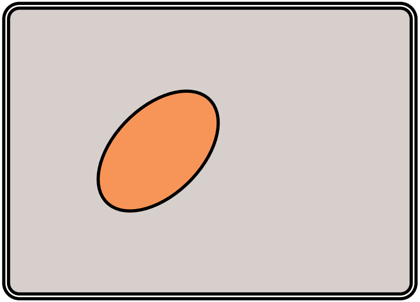
The ingredients

- ***Awareness-of*** and ***awareness-that***.
  - Fagin and Halpern (1988): awareness has ***different interpretations***.
  - Dretske (1993): ***Awareness of things*** vs ***awareness of facts***.

The ingredients

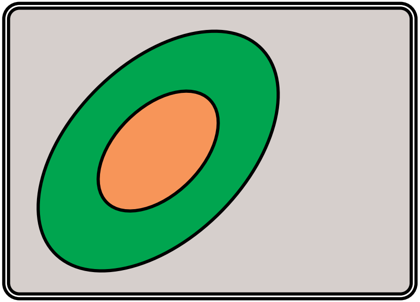
- ***Awareness-of*** and ***awareness-that***.
  - Fagin and Halpern (1988): awareness has *different interpretations*.
  - Dretske (1993): *Awareness of things* vs *awareness of facts*.
- Here:
  - ***Awareness-of*** as *entertaining* ('*working memory*'), not implying any attitude in favour or against.
  - ***Awareness-that*** as *acknowledgement* or *acceptance*.

Combined Diagram



(5) Awareness-that

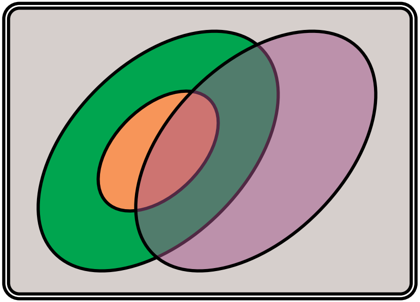
Combined Diagram



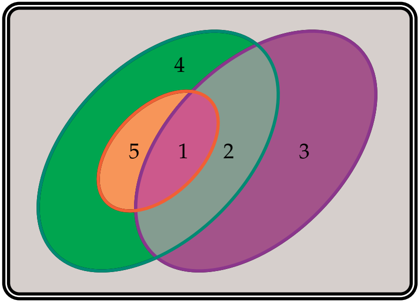
(5) Awareness-that

(4) 'Implicit' Awareness-that

Combined Diagram



Combined Diagram



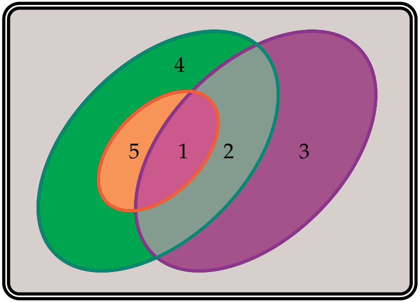
(5) Awareness-that

(4) 'Implicit' Awareness-that (not in working memory)

(3) Awareness-of

(2) Aware-of not aware-that, but deducible

Combined Diagram



(5) Awareness-that (not in working memory)

(4) 'Implicit' Awareness-that (not in working memory)

(3) Awareness-of

(2) Aware-of not aware-that, but deducible

(1) Explicit knowledge (aware-of and aware-that)

Explicit knowledge as awareness *of* + awareness *that*

Awareness neighbourhood model (ANM)

Definition (Awareness neighbourhood model (*ANM*))

Let $\mathbf{P}$ be a set of atoms. An *ANM* is a tuple $M = \langle W, N, V, A \rangle$ where

- $W \neq \emptyset$
- $N : W \rightarrow \wp(\wp(W))$
- $V : \mathbf{P} \rightarrow \wp(W)$
- $A \subseteq \mathbf{P}$

Awareness neighbourhood model (ANM)

Definition (Awareness neighbourhood model (*ANM*))

Let $\mathbf{P}$ be a set of atoms. An *ANM* is a tuple $M = \langle W, N, V, A \rangle$ where

- $W \neq \emptyset$
- $N : W \to \wp(\wp(W))$
- $V : \mathbf{P} \to \wp(W)$
- $A \subseteq \mathbf{P}$

- ***Awareness-that***: (local) neighbourhood function N.

Awareness neighbourhood model (ANM)

Definition (Awareness neighbourhood model (*ANM*))

Let $\mathbf{P}$ be a set of atoms. An *ANM* is a tuple $M = \langle W, N, V, A \rangle$ where

- $W \neq \emptyset$
- $N : W \to \wp(\wp(W))$
- $V : \mathbf{P} \to \wp(W)$
- $A \subseteq \mathbf{P}$

- ***Awareness-that***: (local) neighbourhood function N.
- ***Awareness-of***: (global) set of atoms A.

Language and semantic interpretation (1)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \land \psi \mid \mathbf{A}^{\mathbf{o}}\, \varphi \mid \mathbf{A}^{\mathbf{t}}\, \varphi \mid [*]\, \varphi$$

- $[\![\top]\!]^M := W$,
- $[\![p]\!]^M := V(p)$,
- $[\![\neg\varphi]\!]^M := W \setminus [\![\varphi]\!]^M$,
- $[\![\varphi \land \psi]\!]^M := [\![\varphi]\!]^M \cap [\![\psi]\!]^M$.

Language and semantic interpretation (2)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{A}^{\mathbf{o}}\,\varphi \mid \mathbf{A}^{\mathbf{t}}\,\varphi \mid [*]\,\varphi$$

- $[\![\mathbf{A}^{\mathbf{o}}\,\varphi]\!]^{M} := \begin{cases} W & \text{if } \operatorname{atm}(\varphi) \subseteq A \\ \varnothing & \text{otherwise} \end{cases},$

Language and semantic interpretation (2)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{A}^{\mathbf{o}}\,\varphi \mid \mathbf{A}^{\mathbf{t}}\,\varphi \mid [*]\,\varphi$$

- $[\![\mathbf{A}^{\mathbf{o}}\,\varphi]\!]^{M} := \begin{cases} W & \text{if } \operatorname{atm}(\varphi) \subseteq A \\ \varnothing & \text{otherwise} \end{cases},$
- $[\![\mathbf{A}^{\mathbf{t}}\,\varphi]\!]^{M} := \{ w \in W \mid [\![\varphi]\!]^{M} \in N(w) \}.$

Language and semantic interpretation (3)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{A}^{\mathbf{o}}\,\varphi \mid \mathbf{A}^{\mathbf{t}}\,\varphi \mid [*]\,\varphi$$

Given $M = \langle W, N, V, A\rangle$, define $M^* = \langle W, N^*, V, A\rangle$ with

$$N^*(w) := \left\{U \subseteq W \mid \bigcap N(w) \subseteq U\right\}$$

Language and semantic interpretation (3)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathrm{A}^{\mathrm{o}}\, \varphi \mid \mathrm{A}^{\mathrm{t}}\, \varphi \mid [*]\, \varphi$$

Given $M = \langle W, N, V, A \rangle$, define $M^* = \langle W, N^*, V, A \rangle$ with

$$N^*(w) := \left\{ U \subseteq W \mid \bigcap N(w) \subseteq U \right\}$$

Then

$$[\![[*]\, \varphi]\!]^M := [\![\varphi]\!]^{M^*}$$

Language and semantic interpretation (3)

Definition (Language $\mathcal{L}$)

$$\varphi, \psi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{A}^{\mathbf{o}}\,\varphi \mid \mathbf{A}^{\mathbf{t}}\,\varphi \mid [*]\,\varphi$$

Given $M = \langle W, N, V, A \rangle$, define $M^* = \langle W, N^*, V, A \rangle$ with

$$N^*(w) := \left\{ U \subseteq W \mid \bigcap N(w) \subseteq U \right\}$$

Then

$$[\![[*]\,\varphi]\!]^M := [\![\varphi]\!]^{M^*}$$

The concepts of ***satisfiability*** and ***validity*** are defined as usual.

Explicit knowledge as awareness *of* + awareness *that*

The concepts of Knowledge

Aware knowledge

- ***Explicit knowledge:*** $\mathrm{K}_{Ex}\,\varphi := \mathrm{A}^{\mathrm{o}}\,\varphi \wedge \mathrm{A}^{\mathrm{t}}\,\varphi$

The concepts of Knowledge

Aware knowledge

- ***Explicit knowledge:*** $\mathrm{K}_{Ex}\,\varphi := \mathrm{A}^{\mathrm{o}}\,\varphi \wedge \mathrm{A}^{\mathrm{t}}\,\varphi$
- ***Implicit knowledge:*** $\mathrm{K}_{Im}\,\varphi := \mathrm{A}^{\mathrm{o}}\,\varphi \wedge [*]\,\mathrm{A}^{\mathrm{t}}\,\varphi$

The concepts of Knowledge

Aware knowledge

- *Explicit knowledge:* $\mathrm{K}_{Ex}\,\varphi := \mathrm{A^o}\,\varphi \wedge \mathrm{A^t}\,\varphi$
- *Implicit knowledge:* $\mathrm{K}_{Im}\,\varphi := \mathrm{A^o}\,\varphi \wedge [*]\,\mathrm{A^t}\,\varphi$

Unaware knowledge

- *'Disassociated' knowledge:* $\mathrm{K}^{-\mathrm{o}}_{Ex}\,\varphi := \neg\,\mathrm{A^o}\,\varphi \wedge \mathrm{A^t}\,\varphi$

The concepts of Knowledge

Aware knowledge

- *Explicit knowledge:* $\mathrm{K}_{Ex}\,\varphi := \mathrm{A}^{\mathrm{o}}\,\varphi \wedge \mathrm{A}^{\mathrm{t}}\,\varphi$
- *Implicit knowledge:* $\mathrm{K}_{Im}\,\varphi := \mathrm{A}^{\mathrm{o}}\,\varphi \wedge [*]\,\mathrm{A}^{\mathrm{t}}\,\varphi$

Unaware knowledge

- *'Disassociated' knowledge:* $\mathrm{K}_{Ex}^{-\mathrm{o}}\,\varphi := \neg\,\mathrm{A}^{\mathrm{o}}\,\varphi \wedge \mathrm{A}^{\mathrm{t}}\,\varphi$
- *'Unreachable' knowledge:* $\mathrm{K}_{Im}^{-\mathrm{o}}\,\varphi := \neg\,\mathrm{A}^{\mathrm{o}}\,\varphi \wedge [*]\,\mathrm{A}^{\mathrm{t}}\,\varphi$

Explicit knowledge as awareness *of* + awareness *that*

Properties of Awareness-of (A^o)

- The agent is aware-of the **concept of truth**: $\Vdash A^o \top$

Properties of Awareness-of (A^o)

- The agent is aware-of the **concept of truth**: $\Vdash \mathbf{A^o}\, \top$
  But, $\Vdash \varphi$ does not imply $\Vdash \mathbf{A^o}\, \varphi$

Properties of Awareness-of (A^o)

- The agent is aware-of the **concept of truth**: $\Vdash \mathbf{A^o}\, \top$
  But, $\Vdash \varphi$ does not imply $\Vdash \mathbf{A^o}\, \varphi$
- Since A^o is defined as a set of atomic propositions, it is **closed under subformulas and superformulas**:

Properties of Awareness-of (A^o)

- The agent is aware-of the **concept of truth**: $\Vdash A^o \top$
  But, $\Vdash \varphi$ does not imply $\Vdash A^o \varphi$
- Since A^o is defined as a set of atomic propositions, it is **closed under subformulas and superformulas**:

$$\Vdash A^o \neg\varphi \leftrightarrow A^o \varphi$$

$$\Vdash A^o(\varphi \wedge \psi) \leftrightarrow (A^o \varphi \wedge A^o \psi)$$

$$\Vdash A^o A^o \varphi \leftrightarrow A^o \varphi$$

$$\Vdash A^o A^t \varphi \leftrightarrow A^o \varphi$$

$$\Vdash A^o [*] \varphi \leftrightarrow A^o \varphi$$

Properties of Awareness-that (A^t)

- A^t is what appears in $N(w)$. This pureley semantic concept is **closed under logical equivalence** (some kind of omniscience):
  $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash A^t \varphi \leftrightarrow A^t \psi$

Properties of Awareness-that (A^t)

- A^t is what appears in $N(w)$. This pureley semantic concept is **closed under logical equivalence** (some kind of omniscience):
  $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash A^t \varphi \leftrightarrow A^t \psi$
- But it is the only closure property, since
  $\Vdash \varphi$ does not imply $\Vdash A^t \varphi$
  $\nVdash (A^t \varphi \wedge A^t \psi) \rightarrow A^t(\varphi \wedge \psi)$
  $\nVdash A^t(\varphi \wedge \psi) \rightarrow A^t \varphi$ and $\nVdash A^t(\varphi \wedge \psi) \rightarrow A^t \psi$

Properties of Awareness-that (A^{t})

- A^{t} is what appears in $N(w)$. This pureley semantic concept is **closed under logical equivalence** (some kind of omniscience):
  $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash \mathbf{A}^{\mathbf{t}}\varphi \leftrightarrow \mathbf{A}^{\mathbf{t}}\psi$
- But it is the only closure property, since
  $\Vdash \varphi$ does not imply $\Vdash \mathbf{A}^{\mathbf{t}}\varphi$
  $\nVdash (\mathbf{A}^{\mathbf{t}}\varphi \wedge \mathbf{A}^{\mathbf{t}}\psi) \rightarrow \mathbf{A}^{\mathbf{t}}(\varphi \wedge \psi)$
  $\nVdash \mathbf{A}^{\mathbf{t}}(\varphi \wedge \psi) \rightarrow \mathbf{A}^{\mathbf{t}}\varphi$ and $\nVdash \mathbf{A}^{\mathbf{t}}(\varphi \wedge \psi) \rightarrow \mathbf{A}^{\mathbf{t}}\psi$
- Hence, A^{t} is not closed under logical consequence:
  $\nVdash \mathbf{A}^{\mathbf{t}}(\varphi \rightarrow \psi) \rightarrow (\mathbf{A}^{\mathbf{t}}\varphi \rightarrow \mathbf{A}^{\mathbf{t}}\psi)$

Awareness-of and Awareness-that

In contrast to what happens in *Awareness Logic* by Fagin and Halpern, where $\Vdash A\varphi \rightarrow \Box A\varphi$, with a global awareness set, we do not obtain this result, thanks to the different concepts of awareness we defined.

Recall that **awareness-of** is a global notion and **awareness-that** is locally defined.

Awareness-of and Awareness-that

In contrast to what happens in *Awareness Logic* by Fagin and Halpern, where $\Vdash A\varphi \rightarrow \Box A\varphi$, with a global awareness set, we do not obtain this result, thanks to the different concepts of awareness we defined.

Recall that **awareness-of** is a global notion and **awareness-that** is locally defined.

Thus, analogous properties do not hold:

- $\nVdash \mathbf{A}^{\mathbf{o}}\varphi \rightarrow \mathbf{A}^{\mathbf{t}}\mathbf{A}^{\mathbf{o}}\varphi$
- $\nVdash \neg\mathbf{A}^{\mathbf{o}}\varphi \rightarrow \mathbf{A}^{\mathbf{t}}\neg\mathbf{A}^{\mathbf{o}}\varphi$

Properties of Explicit Knowledge (K_{Ex})

Recall that $\mathbf{K}_{Ex}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \wedge \mathbf{A}^{\mathbf{t}}\,\varphi$. This definition has the following **consequences**:

Properties of Explicit Knowledge (K_{Ex})

Recall that $\mathbf{K}_{Ex}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \wedge \mathbf{A}^{\mathbf{t}}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{o}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{t}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$

Properties of Explicit Knowledge (K_{Ex})

Recall that $\mathsf{K}_{Ex}\,\varphi := \mathsf{A}^{\mathsf{o}}\,\varphi \wedge \mathsf{A}^{\mathsf{t}}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash \mathsf{K}_{Ex}\,\varphi$; nor $\Vdash \mathsf{A}^{\mathsf{o}}\,\varphi \rightarrow \mathsf{K}_{Ex}\,\varphi$; nor $\Vdash \mathsf{A}^{\mathsf{t}}\,\varphi \rightarrow \mathsf{K}_{Ex}\,\varphi$
- About **logical equivalence**: $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash \mathsf{K}_{Ex}\,\varphi \leftrightarrow \mathsf{K}_{Ex}\,\psi$

Properties of Explicit Knowledge ($\mathbf{K}_{Ex}$)

Recall that $\mathbf{K}_{Ex}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \wedge \mathbf{A}^{\mathbf{t}}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{o}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{t}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash \mathbf{K}_{Ex}\,\varphi \leftrightarrow \mathbf{K}_{Ex}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (\mathbf{K}_{Ex}\,\varphi \wedge \mathbf{A}^{\mathbf{o}}\,\psi) \rightarrow \mathbf{K}_{Ex}\,\psi$

Properties of Explicit Knowledge (K_{Ex})

Recall that $K_{Ex}\,\varphi := A^{o}\,\varphi \wedge A^{t}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash K_{Ex}\,\varphi$; nor $\Vdash A^{o}\,\varphi \rightarrow K_{Ex}\,\varphi$; nor $\Vdash A^{t}\,\varphi \rightarrow K_{Ex}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash K_{Ex}\,\varphi \leftrightarrow K_{Ex}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (K_{Ex}\,\varphi \wedge A^{o}\,\psi) \rightarrow K_{Ex}\,\psi$
- About **Modus Ponens**:
  $\nVdash K_{Ex}(\varphi \rightarrow \psi) \rightarrow (K_{Ex}\,\varphi \rightarrow K_{Ex}\,\psi)$

Properties of Explicit Knowledge (K_{Ex})

Recall that $\mathbf{K}_{Ex}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \wedge \mathbf{A}^{\mathbf{t}}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{o}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$; nor $\Vdash \mathbf{A}^{\mathbf{t}}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash \mathbf{K}_{Ex}\,\varphi \leftrightarrow \mathbf{K}_{Ex}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (\mathbf{K}_{Ex}\,\varphi \wedge \mathbf{A}^{\mathbf{o}}\,\psi) \rightarrow \mathbf{K}_{Ex}\,\psi$
- About **Modus Ponens**:
  $\nVdash \mathbf{K}_{Ex}(\varphi \rightarrow \psi) \rightarrow (\mathbf{K}_{Ex}\,\varphi \rightarrow \mathbf{K}_{Ex}\,\psi)$
  $\nVdash \mathbf{K}_{Ex}(\varphi \rightarrow \psi) \rightarrow ((\mathbf{K}_{Ex}\,\varphi \wedge \mathbf{A}^{\mathbf{o}}\,\psi) \rightarrow \mathbf{K}_{Ex}\,\psi)$

Properties of Explicit Knowledge (K_{Ex})

Recall that $K_{Ex}\,\varphi := A^{o}\,\varphi \land A^{t}\,\varphi$. This definition has the following **consequences**:

- About **validities**: $\Vdash \varphi$ implies neither $\Vdash K_{Ex}\,\varphi$; nor $\Vdash A^{o}\,\varphi \rightarrow K_{Ex}\,\varphi$; nor $\Vdash A^{t}\,\varphi \rightarrow K_{Ex}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash K_{Ex}\,\varphi \leftrightarrow K_{Ex}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (K_{Ex}\,\varphi \land A^{o}\,\psi) \rightarrow K_{Ex}\,\psi$
- About **Modus Ponens**:
  $\nVdash K_{Ex}(\varphi \rightarrow \psi) \rightarrow (K_{Ex}\,\varphi \rightarrow K_{Ex}\,\psi)$
  $\nVdash K_{Ex}(\varphi \rightarrow \psi) \rightarrow ((K_{Ex}\,\varphi \land A^{o}\,\psi) \rightarrow K_{Ex}\,\psi)$
  But, $\Vdash K_{Ex}(\varphi \rightarrow \psi) \rightarrow ((K_{Ex}\,\varphi \land A^{t}\,\psi) \rightarrow K_{Ex}\,\psi)$

Explicit knowledge as awareness *of* + awareness *that*

Awareness-that after full deductive inference ($[*]\,\mathrm{A}^t$)

The closure operation $[*]$ *makes* A^t *behave as* $\Box$ *in relational models.*

Awareness-that after full deductive inference ($[*]\,\mathrm{A}^{\mathrm{t}}$)

The closure operation $[*]$ *makes* A^{t} *behave as* $\Box$ *in relational models.*

Some results on:

Awareness-that after full deductive inference ($[*]\,\mathrm{A^t}$)

The closure operation $[*]$ *makes* $\mathrm{A^t}$ *behave as* $\Box$ *in relational models.*

Some results on:
Validities: $\Vdash \varphi$ implies $\Vdash [*]\,\mathbf{A^t}\,\varphi$

Awareness-that after full deductive inference ($[*]\,\mathrm{A}^{\mathrm{t}}$)

The closure operation $[*]$ makes A^{t} behave as $\Box$ in relational models.

Some results on:

Validities: $\Vdash \varphi$ implies $\Vdash [*]\,\mathrm{A}^{\mathrm{t}}\,\varphi$

Conjunction intr.: $\Vdash ([*]\,\mathrm{A}^{\mathrm{t}}\,\varphi \wedge [*]\,\mathrm{A}^{\mathrm{t}}\,\psi) \rightarrow [*]\,\mathrm{A}^{\mathrm{t}}(\varphi \wedge \psi)$

Awareness-that after full deductive inference ($[*]\,\mathrm{A}^{\mathrm{t}}$)

The closure operation $[*]$ *makes* A^{t} *behave as* $\Box$ *in relational models.*

Some results on:

Validities: $\Vdash \varphi$ implies $\Vdash [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$

Conjunction intr.: $\Vdash ([*]\,\mathbf{A}^{\mathbf{t}}\,\varphi \land [*]\,\mathbf{A}^{\mathbf{t}}\,\psi) \rightarrow [*]\,\mathbf{A}^{\mathbf{t}}(\varphi \land \psi)$

elim.: $\Vdash [*]\,\mathbf{A}^{\mathbf{t}}(\varphi \land \psi) \rightarrow [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$ and $\Vdash [*]\,\mathbf{A}^{\mathbf{t}}(\varphi \land \psi) \rightarrow [*]\,\mathbf{A}^{\mathbf{t}}\,\psi$

Awareness-that after full deductive inference ($[*]\,\mathrm{A}^t$)

The closure operation $[*]$ *makes* A^t *behave as* $\Box$ *in relational models.*

Some results on:

Validities: $\Vdash \varphi$ implies $\Vdash [*]\,\mathbf{A}^t\,\varphi$

Conjunction intr.: $\Vdash ([*]\,\mathbf{A}^t\,\varphi \land [*]\,\mathbf{A}^t\,\psi) \rightarrow [*]\,\mathbf{A}^t(\varphi \land \psi)$

elim.: $\Vdash [*]\,\mathbf{A}^t(\varphi \land \psi) \rightarrow [*]\,\mathbf{A}^t\,\varphi$ and $\Vdash [*]\,\mathbf{A}^t(\varphi \land \psi) \rightarrow [*]\,\mathbf{A}^t\,\psi$

Closure under MP: $\Vdash [*]\,(\mathbf{A}^t(\varphi \rightarrow \psi) \rightarrow (\mathbf{A}^t\,\varphi \rightarrow \mathbf{A}^t\,\psi))$

Awareness-that after full deductive inference ($[*]\,A^t$)

The closure operation $[*]$ makes A^t behave as $\Box$ in relational models.

Some results on:

Validities: $\Vdash \varphi$ implies $\Vdash [*]\,A^t\varphi$

Conjunction intr.: $\Vdash ([*]\,A^t\varphi \wedge [*]\,A^t\psi) \rightarrow [*]\,A^t(\varphi \wedge \psi)$

elim.: $\Vdash [*]\,A^t(\varphi \wedge \psi) \rightarrow [*]\,A^t\varphi$ and $\Vdash [*]\,A^t(\varphi \wedge \psi) \rightarrow [*]\,A^t\psi$

Closure under MP: $\Vdash [*]\,(A^t(\varphi \rightarrow \psi) \rightarrow (A^t\varphi \rightarrow A^t\psi))$

$\Vdash [*]\,A^t(\varphi \rightarrow \psi) \rightarrow ([*]\,A^t\varphi \rightarrow [*]\,A^t\psi)$

Awareness-that after full deductive inference ($[*]\,A^t$)

The closure operation $[*]$ makes A^t behave as $\Box$ in relational models.

Some results on:

Validities: $\Vdash \varphi$ implies $\Vdash [*]\,\mathrm{A}^t\,\varphi$

Conjunction intr.: $\Vdash ([*]\,\mathrm{A}^t\,\varphi \land [*]\,\mathrm{A}^t\,\psi) \rightarrow [*]\,\mathrm{A}^t(\varphi \land \psi)$

elim.: $\Vdash [*]\,\mathrm{A}^t(\varphi \land \psi) \rightarrow [*]\,\mathrm{A}^t\,\varphi$ and $\Vdash [*]\,\mathrm{A}^t(\varphi \land \psi) \rightarrow [*]\,\mathrm{A}^t\,\psi$

Closure under MP: $\Vdash [*]\,(\mathrm{A}^t(\varphi \rightarrow \psi) \rightarrow (\mathrm{A}^t\,\varphi \rightarrow \mathrm{A}^t\,\psi))$

$\Vdash [*]\,\mathrm{A}^t(\varphi \rightarrow \psi) \rightarrow ([*]\,\mathrm{A}^t\,\varphi \rightarrow [*]\,\mathrm{A}^t\,\psi)$

Aware-that awareness-of is the case:

$\Vdash [*](\mathrm{A}^o\,\varphi \rightarrow \mathrm{A}^t\,\mathrm{A}^o\,\varphi)$ and $\Vdash [*](\neg\,\mathrm{A}^o\,\varphi \rightarrow \mathrm{A}^t\,\neg\,\mathrm{A}^o\,\varphi)$

Properties of Implicit Knowledge (K_{Im})

Recall that $\mathbf{K}_{Im}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \land [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$. This has the following consequences:

Properties of Implicit Knowledge ($\mathbf{K}_{Im}$)

Recall that $\mathbf{K}_{Im}\,\varphi := \mathbf{A}^{\mathbf{o}}\,\varphi \wedge [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$. This has the following consequences:

- About **validities**: $\Vdash \varphi$ does not imply $\Vdash \mathbf{K}_{Im}\,\varphi$
  But, $\Vdash \varphi$ implies $\Vdash \mathbf{A}^{\mathbf{o}}\,\varphi \rightarrow \mathbf{K}_{Im}\,\varphi$

Properties of Implicit Knowledge (K_{Im})

Recall that $K_{Im}\,\varphi := A^{o}\,\varphi \wedge [*]\,A^{t}\,\varphi$. This has the following consequences:

- About **validities**: $\Vdash \varphi$ does not imply $\Vdash K_{Im}\,\varphi$
  But, $\Vdash \varphi$ implies $\Vdash A^{o}\,\varphi \rightarrow K_{Im}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash K_{Im}\,\varphi \leftrightarrow K_{Im}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (K_{Im}\,\varphi \wedge A^{o}\,\psi) \rightarrow K_{Im}\,\psi$

Properties of Implicit Knowledge (K_{Im})

Recall that $K_{Im}\,\varphi := A^{o}\,\varphi \wedge [*]\,A^{t}\,\varphi$. This has the following consequences:

- About **validities**: $\Vdash \varphi$ does not imply $\Vdash K_{Im}\,\varphi$
  But, $\Vdash \varphi$ implies $\Vdash A^{o}\,\varphi \rightarrow K_{Im}\,\varphi$
- About **logical equivalence**:
  $\Vdash \varphi \leftrightarrow \psi$ does not imply $\Vdash K_{Im}\,\varphi \leftrightarrow K_{Im}\,\psi$
  But, $\Vdash \varphi \leftrightarrow \psi$ implies $\Vdash (K_{Im}\,\varphi \wedge A^{o}\,\psi) \rightarrow K_{Im}\,\psi$
- About **Modus Ponens**:
  $\Vdash K_{Im}(\varphi \rightarrow \psi) \rightarrow (K_{Im}\,\varphi \rightarrow K_{Im}\,\psi)$
  Thus, $\Vdash (K_{Im}\,\varphi \wedge K_{Im}\,\psi) \rightarrow K_{Im}(\varphi \wedge \psi)$; and also
  $\Vdash K_{Im}(\varphi \wedge \psi) \rightarrow K_{Im}\,\varphi$ and $\Vdash K_{Im}(\varphi \wedge \psi) \rightarrow K_{Im}\,\psi$

Explicit knowledge as awareness *of* + awareness *that*

The Moorean Phenomena

- *'Implicit is not always Explicit':* $\nvDash \mathbf{K}_{Ex}\, \varphi \rightarrow \mathbf{K}_{Im}\, \varphi$

The Moorean Phenomena

- *'Implicit is not always Explicit':* $\nvdash \mathbf{K}_{Ex}\,\varphi \rightarrow \mathbf{K}_{Im}\,\varphi$
  What the agent has acknowledged as true does not need to hold after the closure operation. Thus,

The Moorean Phenomena

- *'Implicit is not always Explicit':* $\nVdash \mathbf{K}_{Ex}\,\varphi \rightarrow \mathbf{K}_{Im}\,\varphi$
  What the agent has acknowledged as true does not need to hold after the closure operation. Thus,
- $\nVdash \mathbf{A}^{t}\,\varphi \rightarrow [*]\,\mathbf{A}^{t}\,\varphi$.

The Moorean Phenomena

- *'Implicit is not always Explicit'*: $\nVdash \mathbf{K}_{Ex}\,\varphi \rightarrow \mathbf{K}_{Im}\,\varphi$
  What the agent has acknowledged as true does not need to hold after the closure operation. Thus,
- $\nVdash \mathbf{A}^{\mathbf{t}}\,\varphi \rightarrow [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$. Take $\varphi := \neg\,\mathbf{A}^{\mathbf{t}}\,q$,
  then $\mathbf{A}^{\mathbf{t}}\,\neg\,\mathbf{A}^{\mathbf{t}}\,q$ has a similar effect as a ***Moore sentence***, stating "the agent is aware that it is the case that she is not aware that q is the case".

The Moorean Phenomena

- *'Implicit is not always Explicit':* $\nvDash \mathbf{K}_{Ex}\,\varphi \rightarrow \mathbf{K}_{Im}\,\varphi$
  What the agent has acknowledged as true does not need to hold after the closure operation. Thus,
- $\nvDash \mathbf{A}^{\mathbf{t}}\,\varphi \rightarrow [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$. Take $\varphi := \neg\,\mathbf{A}^{\mathbf{t}}\,q$,
  then $\mathbf{A}^{\mathbf{t}}\,\neg\,\mathbf{A}^{\mathbf{t}}\,q$ has a similar effect as a ***Moore sentence***, stating "the agent is aware that it is the case that she is not aware that q is the case".
- While '$\mathrm{A}^{\mathrm{t}}\,\neg\,\mathrm{A}^{\mathrm{t}}\,q$' is true at M, it will not be the case at M^*, since its truthset has shrunk after the operation.

The Moorean Phenomena

- *'Implicit is not always Explicit':* $\nVdash \mathbf{K}_{Ex}\,\varphi \to \mathbf{K}_{Im}\,\varphi$
  What the agent has acknowledged as true does not need to hold after the closure operation. Thus,
- $\nVdash \mathbf{A}^{\mathbf{t}}\,\varphi \to [*]\,\mathbf{A}^{\mathbf{t}}\,\varphi$. Take $\varphi := \neg\,\mathbf{A}^{\mathbf{t}}\,q$, then $\mathbf{A}^{\mathbf{t}}\,\neg\,\mathbf{A}^{\mathbf{t}}\,q$ has a similar effect as a ***Moore sentence***, stating "the agent is aware that it is the case that she is not aware that q is the case".
- While '$\mathrm{A}^{\mathrm{t}}\,\neg\,\mathrm{A}^{\mathrm{t}}\,q$' is true at M, it will not be the case at M^*, since its truthset has shrunk after the operation.
- Though, $\Vdash \varphi \to [*]\,\varphi$ implies $\Vdash \mathbf{K}_{Ex}\,\varphi \to \mathbf{K}_{Im}\,\varphi$

Explicit knowledge as awareness *of* + awareness *that*

Other alternatives for representing our basic concepts

For representing Awareness-of:

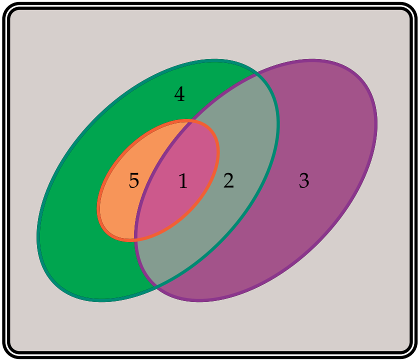
- Concept of ***topics*** in Berto and Hawke (2018) (cf. Berto 2018). (A *topic* being what the sentence is *about*.)
- The ***issue relation*** in, e.g., Grossi (2009), van Benthem and Minică (2012), Baltag et al. (2018). (Equivalence relation that creates partitions of the domain in relational model.)

For representing Awareness-that:

- ***Explicit knowledge*** in proposals not incorporating the notion of *awareness*, e.g., Konolige 1984, Levesque 1984, Duc 1997, Artemov and Nogina 2005, Jago 2009, Velázquez-Quesada 2013.
- Alternatives where the knowledge/belief relies on ***evidences*** (van Benthem and Pacuit 2011, Özgün 2017) and ***arguments*** (Shi et al. 2018a, 2017, 2018b).

Explicit knowledge as awareness *of* + awareness *that*

Recall: combined diagram



(5) Awareness-that (not in working memory)

(4) 'Implicit' Awareness-that (not in working memory)

(3) Awareness-of

(2) Aware-of not aware-that, but deducible

(1) Explicit knowledge (aware-of and aware-that)

Action: Becoming Aware-of $[+\chi]$ (1)

Definition (The becoming aware-of operation)

Given $M = \langle W, N, V, A \rangle$ *and* $\boldsymbol{M^{+\chi} = \langle W, N, V, A^{+\chi} \rangle}$, *we have*

$$\boldsymbol{A^{+\chi} = A \cup \mathrm{atm}(\chi)}$$

Action: Becoming Aware-of $[+\chi]$ (1)

Definition (The becoming aware-of operation)

Given $M = \langle W, N, V, A\rangle$ *and* $\boldsymbol{M^{+\chi} = \langle W, N, V, A^{+\chi}\rangle}$, *we have*

$$\boldsymbol{A^{+\chi} = A \cup \mathrm{atm}(\chi)}$$

Then, we define $[\![[+\chi]\,\varphi]\!]^{M} = [\![\varphi]\!]^{M^{+\chi}}$ *and extend the language* $\mathcal{L}$ *with* $[+\chi]\,\varphi$, *read as* ***after the agent becomes aware-of*** χ, φ ***is the case***.

Action: Becoming Unaware-of $[-\chi]$

Definition (The becoming unaware-of operation)

Given $M = \langle W, N, V, A \rangle$ *and* $\boldsymbol{M^{-\chi} = \langle W, N, V, A^{-\chi} \rangle}$*, we have*

$$\boldsymbol{A^{-\chi} = A \setminus \mathrm{atm}(\chi)}$$

Action: Becoming Unaware-of $[-\chi]$

Definition (The becoming unaware-of operation)

Given $M = \langle W, N, V, A \rangle$ *and* $\mathbf{M^{-\chi} = \langle W, N, V, A^{-\chi} \rangle}$, *we have*

$$\mathbf{A^{-\chi} = A \setminus \mathrm{atm}(\chi)}$$

Then, we define $\mathbf{[\![[-\chi]\, \varphi]\!]^{M} = [\![\varphi]\!]^{M^{-\chi}}}$ *and extend the language* $\mathcal{L}$ *with* $\mathbf{[-\chi]\, \varphi}$, *read as* ***after the agent becomes unaware-of*** χ, φ ***will be the case.***

Action: Becoming Unaware-of $[-\chi]$

Definition (The becoming unaware-of operation)

Given $M = \langle W, N, V, A \rangle$ *and* $M^{-\chi} = \langle W, N, V, A^{-\chi} \rangle$*, we have*

$$A^{-\chi} = A \setminus \mathrm{atm}(\chi)$$

Then, we define $[\![[-\chi]\,\varphi]\!]^M = [\![\varphi]\!]^{M^{-\chi}}$ *and extend the language* $\mathcal{L}$ *with* $[-\chi]\,\varphi$*, read as* ***after the agent becomes unaware-of*** χ***,*** φ ***will be the case.***

Alternative definition: *weak* **becoming unawarene-of**

- $[\![[-'Q]\,\varphi]\!]^M = [\![\varphi]\!]^{M^{-Q}}$
- $[\![[-'\chi]\,\varphi]\!]^M = [\![\bigwedge_{\{Q \subseteq \mathrm{atm}(\chi) \mid Q \neq \emptyset\}} [-Q]\,\varphi]\!]^M$
- $[\![\langle -\chi \rangle\,\varphi]\!]^M = [\![\bigvee_{\{Q \subseteq \mathrm{atm}(\chi) \mid Q \neq \emptyset\}} [-Q]\,\varphi]\!]^M$

Action: Deductive Inference (Modus Ponens step) $[\eta \rightarrow \chi]$

Definition (The deductive inference operation)

For $\eta, \chi, \varphi \in \mathcal{L}$, $\mathrm{atm}(\eta \rightarrow \chi) \subseteq A$, *and* $\boldsymbol{M = \langle W, N, V, A \rangle}$, *we have* $\boldsymbol{M^{\eta \rightarrow \chi} = \langle W, N^{\eta \rightarrow \chi}, V, A \rangle}$ *where for any* $w \in W$:

$$N^{\eta \rightarrow \chi}(w) = \begin{cases} N(w) \cup [\![\chi]\!]^M & \text{if } \left\{[\![(\eta \rightarrow \chi)]\!]^M, [\![\eta]\!]^M\right\} \subseteq N(w) \\ N(w) & \text{otherwise} \end{cases}$$

Action: Deductive Inference (Modus Ponens step) $[\eta \to \chi]$

Definition (The deductive inference operation)

For $\eta, \chi, \varphi \in \mathcal{L}$, $\text{atm}(\eta \to \chi) \subseteq A$, *and* $\mathbf{M} = \langle W, N, V, A \rangle$, *we have* $\mathbf{M}^{\eta \to \chi} = \langle W, N^{\eta \to \chi}, V, A \rangle$ *where for any* $w \in W$:

$$N^{\eta \to \chi}(w) = \begin{cases} N(w) \cup [\![\chi]\!]^M & \text{if } \left\{[\![(\eta \to \chi)]\!]^M, [\![\eta]\!]^M\right\} \subseteq N(w) \\ N(w) & \text{otherwise} \end{cases}$$

Then, we define $[\![[\eta \to \chi]\,\varphi]\!]^M = [\![\varphi]\!]^{M^{\eta \to \chi}}$ *and extend the language* $\mathcal{L}$ *with* $[\eta \to \chi]\,\varphi$, *read as* ***after the agent performs a deductive inference from*** $\eta \to \chi$ ***and*** η ***holds,*** φ ***is the case.***

Action: Forgetting $[\backslash\chi]$

Definition (The forgetting operation)

For $\chi \in \mathcal{L}$ *such that* $\mathrm{atm}(\chi) \subseteq A$*, we have* $M = \langle W, N, V, A \rangle$ *and* $\boldsymbol{M^{\backslash\chi} = \langle W, N^{\backslash\chi}, V, A \rangle}$ *where for* $w \in W$*:*

$$\boldsymbol{N^{\backslash\chi}(w) = N(w) \setminus [\![\chi]\!]^M}$$

Action: Forgetting $[\backslash\chi]$

Definition (The forgetting operation)

For $\chi \in \mathcal{L}$ such that $\mathrm{atm}(\chi) \subseteq A$, we have $M = \langle W, N, V, A \rangle$ and $M^{\backslash\chi} = \left\langle W, N^{\backslash\chi}, V, A \right\rangle$ where for $w \in W$:

$$N^{\backslash\chi}(w) = N(w) \setminus [\![\chi]\!]^M$$

*Then, we define $[\![[\backslash\chi]\,\varphi]\!]^M = [\![\varphi]\!]^{M^{\backslash\chi}}$ and extend the language $\mathcal{L}$ with $[\backslash\chi]\,\varphi$, read as **after the agent forgets χ, φ is the case.***

Explicit knowledge as awareness *of* + awareness *that*

Summarizing

- *Awareness-of* and *awareness-that* as primitive concepts defining ***explicit knowledge***.
- A *semantic model*; defined the involved notions.
- *Properties* as compared with related approaches (e.g., Hintikka 1962, Konolige 1984, Fagin and Halpern 1988).

Current and Future Work

- More precise comparison with other *semantic alternatives*
- Axiom system.
- Further ***epistemic actions*** like *observation or communication*

Thank you!

¡Muchas gracias!

S. N. Artemov and E. Nogina. Introducing justification into epistemic logic. *Journal of Logic and Computation*, 15(6):1059–1073, 2005. doi: 10.1093/logcom/exi053. URL http://dx.doi.org/10.1093/logcom/exi053.

A. Baltag, R. Boddy, and S. Smets. Group knowledge in interrogative epistemology. In H. van Ditmarsch and G. Sandu, editors, *Jaakko Hintikka*, volume 12 of *Outstanding Contributions to Logic*, pages 131–164. Springer, 2018. ISBN 978-3-319-62863-9. doi: 10.1007/978-3-319-62864-6_5. URL http://www.illc.uva.nl/Research/Publications/Reports/PP-2017-15.text.pdf.

F. Berto. Aboutness in imagination. *Philosophical Studies*, 175(8):1871–1886, Aug. 2018. doi: 10.1007/s11098-017-0937-y. URL http://hdl.handle.net/11245.1/858f32a7-2293-4414-8e8d-7aae16ad5d99.

F. Berto and P. Hawke. Knowability relative to information. *Mind*, 2018. URL https://philpapers.org/go.pl?id=6248898&aid=BERKRTv1. To appear.

F. Dretske. Conscious experience. *Mind*, 102(406):263–283, Apr. 1993. doi: 10.1093/mind/102.406.263.

H. N. Duc. Reasoning about rational, but not logically omniscient, agents. *Journal of Logic and Computation*, 7(5):633–648, 1997. doi: 10.1093/logcom/7.5.633.

R. Fagin and J. Y. Halpern. Belief, awareness, and limited reasoning. *Artificial Intelligence*, 34(1):39–76, 1988. doi: 10.1016/0004-3702(87)90003-8.

D. Grossi. A note on brute vs. institutional facts: Modal logic of equivalence up to a signature. In G. Boella, P. Noriega, G. Pigozzi, and H. Verhagen, editors, *Normative Multi-Agent Systems*, number 09121 in Dagstuhl Seminar Proceedings, Dagstuhl, Germany, 2009. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany. URL http://drops.dagstuhl.de/opus/volltexte/2009/1910.

J. Hintikka. *Knowledge and Belief*. Cornell U.P., Ithaca, 1962. ISBN 1-904987-08-7.

M. Jago. Epistemic logic for rule-based agents. *Journal of Logic, Language and Information*, 18(1):131–158, 2009. doi: 10.1007/s10849-008-9071-8. URL https://doi.org/10.1007/s10849-008-9071-8.

K. Konolige. *A Deduction Model of Belief and its Logics*. PhD thesis, Computer Science Department, Stanford University, Stanford, USA, 1984.

H. J. Levesque. A logic of implicit and explicit belief. In R. J. Brachman, editor, *Proceedings of the National Conference on Artificial Intelligence. Austin, TX, August 6-10, 1984.*, pages 198–202. AAAI Press, 1984. ISBN 0-262-51053-7. URL http://www.aaai.org/Library/AAAI/1984/aaai84-038.php.

A. Özgün. *Evidence in Epistemic Logic: A Topological Perspective*. PhD thesis, Institute for Logic, Language and Computation (ILLC), Universiteit van Amsterdam, Amsterdam, The Netherlands, Oct. 2017. URL http://www.illc.uva.nl/Research/Publications/Dissertations/DS-2017-07.text.pdf. ILLC Dissertation series DS-2017-07.

C. Shi, S. Smets, and F. R. Velázquez-Quesada. Argument-based belief in topological structures. In J. Lang, editor, *Proceedings Sixteenth Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2017, Liverpool, UK, 24-26 July 2017.*, volume 251 of *EPTCS*, pages 489–503, 2017. doi: 10.4204/EPTCS.251.36. URL https://doi.org/10.4204/EPTCS.251.36.

C. Shi, S. Smets, and F. R. Velázquez-Quesada. Beliefs supported by binary arguments. *Journal of Applied Non-Classical Logics*, 2018a. ISSN 1166-3081. doi: 10.1080/11663081.2018.1457254.

C. Shi, S. Smets, and F. R. Velázquez-Quesada. Beliefs based on evidence and argumentation. In L. S. Moss, R. J. G. B. de Queiroz, and M. Martínez, editors, *Logic, Language, Information, and Computation - 25th International Workshop, WoLLIC 2018, Bogota, Colombia, July 24-27, 2018, Proceedings*, volume 10944 of *Lecture Notes in Computer Science*, pages 289–306. Springer, 2018b. ISBN 978-3-662-57668-7. doi: 10.1007/978-3-662-57669-4_17. URL https://doi.org/10.1007/978-3-662-57669-4_17.

J. van Benthem and S. Minică. Toward a dynamic logic of questions. *Journal of Philosophical Logic*, 41(4):633–669, 2012. doi: 10.1007/s10992-012-9233-7.

J. van Benthem and E. Pacuit. Dynamic logics of evidence-based beliefs. *Studia Logica*, 99(1):61–92, 2011. doi: 10.1007/s11225-011-9347-x.

F. R. Velázquez-Quesada. Explicit and implicit knowledge in neighbourhood models. In D. Grossi, O. Roy, and H. Huang, editors, *Logic, Rationality, and Interaction - 4th International Workshop, LORI 2013, Hangzhou, China, October 9-12, 2013, Proceedings*, volume 8196 of *Lecture Notes in Computer Science*, pages 239–252. Springer, 2013. ISBN 978-3-642-40947-9. doi: 10.1007/978-3-642-40948-6_19. URL https://doi.org/10.1007/978-3-642-40948-6_19.