New hyperbolic models and numerical algorithms for Newtonian and general relativistic continuum physics

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Abstract

In the first part of the talk we present high order arbitrary high order accurate (ADER) finite volume and discontinuous Galerkin finite element schemes for the numerical solution of a new unified first order symmetric hyperbolic and thermodynamically compatible (SHTC) formulation of Newtonian continuum physics, including a general description of fluid and solid mechanics as well as electro-magnetic fields in one single system of governing partial differential equations (PDE). The model is based on previous work of Godunov, Peshkov and Romenski (so-called GPR model) on symmetric hyperbolic and thermodynamically compatible systems.

In the second part of the talk, we show a successful extension of the GPR model to general relativity, leading to a novel and unified first order hyperbolic formulation of general relativistic continuum mechanics. The model is able to describe general relativistic solids as well as ideal and non-ideal (viscous) fluids in general relativity. Formal asymptotic expansion of the governing PDE reveals the structure of the viscous stress tensor in the asymptotic relaxation limit. The key features of the new model are its symmetric hyperbolicity and thermodynamical compatibility. The proposed PDE system is causal, covariant and has bounded signal speeds for all involved processes, including dissipative ones. Since the new model also contains elastic solids as a special case, it should be understood as an alternative to existing models for viscous relativistic fluids that are usually derived from kinetic theory and extended irreversible thermodynamics. Our new formulation is particularly well suited for implementation in already existing general relativistic hydrodynamics codes based on the usual 3+1 split. We present numerical results obtained with high order ADER schemes for inviscid and viscous relativistic flows obtained in the stiff relaxation limit of the system, as well as results for solid mechanics.

In the last part of the talk we introduce a new, provably strongly hyperbolic first order reduction of the CCZ4 formalism of the Einstein field equations of general relativity and its solution with high order ADER discontinuous Galerkin finite element schemes.