Construct, Merge, Solve and Adapt for Taxi Sharing

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1 Introduction

Taxis are a quick and reliable mean of transportation, especially in those cities where the public transportation system is very inefficient. However, taxis rarely travel with full capacity, and its impact on traffic congestion and pollution in cities is usually important. For this reason, it is interesting to share taxis instead of traveling alone. We are interested in this paper in finding good solutions in a reasonable time for the taxi sharing problem for the very large instances. In order to do this, we adapt the Construct, Merge, Solve and Adapt (CMSA) algorithm \cite{2} to the taxi sharing problem.

2 Taxi sharing problem

Let us imagine that a group of people in the same place decides to travel to different destinations using taxis. The Taxi Sharing Problem consists in determining the appropriate number of taxis, the assignment of people to taxis and the order in which the taxis must drop the people off, in order to minimize the total monetary cost of the group of people. In our previous paper \cite{1} we explored the use of exact techniques to solve the taxi sharing problem, using the following MILP formulation:

\[
\min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ij},
\]

subject to:

\[
\sum_{i=0}^{n} x_{ij} = 1 \quad \text{for } 1 \leq j \leq n \quad (2)
\]

\[
\sum_{j=0}^{n} x_{ij} = 1 \quad \text{for } 1 \leq i \leq n \quad (3)
\]

\[
y_i - y_j + nx_{ij} \leq n - 1 \quad \text{for } 1 \leq i \neq j \leq n \quad (4)
\]

\[
u_i - u_j + kx_{ij} \leq k - 1 \quad \text{for } 1 \leq i \neq j \leq n \quad (5)
\]
where $x_{ij}$ are binary variables and $u_i$ and $y_i$ are real-valued variables.

The results showed that the exact resolution using CPLEX is appropriate for small, medium-sized and large instances. However, for very large instances, it was not possible to obtain the optimal solution in a reasonable time.

3 CMSA for Taxi Sharing

The Construct, Merge, Solve and Adapt (CMSA) algorithm was designed by Blum et al. [2] and combines a high level heuristic with an exact resolution of small sub-instances of the problem. There are three key ingredients in the algorithm that must be defined for each problem. The algorithm assumes that a solution is a set of components from a larger set $C$. Thus, we need to define what is a component in each problem. In our case a component will be an oriented edge in the graph representing the origin and destinations of the people.

The second ingredient is a method to generate randomized good quality solutions. Our method generates a random permutation of all the destinations. This path is divided in many paths to fulfill the capacity constraint of the taxis (which is 4 people in our case). Next, it applies local search using the swap neighborhood (every pair of destinations is swapped) until a local optimum is found.

Finally, we need in CMSA a MILP formulation for solving the sub-instances of the problem where only the components in $Z_{sub}$ should be included [2]. We start from the formulation in (1)-(5) and we add a new constraint to restrict the edges (components) that appear in any solution. The new constraint is:

$$x_{ij} \leq l_{ij} \quad \text{for } 1 \leq i \neq j \leq n$$

where $l_{ij}$ is 1 if the component $(i,j)$ is included in $Z_{sub}$ and 0 otherwise.

4 Results

For the experiments we run CMSA during 1 minute, which is a reasonable time to assign a very large group of passengers in several taxis. We use the instances of largest size in [1]. We compared our results with those obtained by $p\mu$EA and CPLEX [1], and we observed that the cost obtained by the CMSA algorithm is significantly lower than the cost obtained by $p\mu$EA and in most cases it is near the optimum provided by CPLEX.

References