Design and characterization of a pure phase modulator based on TNLC

Lorena Pérez-Aguilar, Antonio Puerta-Notario, José M. Garrido-Balsells, Miguel del Castillo-Vázquez.

lorenaperezaguilar@ic.uma.es, apn@ic.uma.es, jmgb@ic.uma.es, miguelc@ic.uma.es.

Dpto. de Ingeniería de Comunicaciones . Universidad de Málaga. 29071.

***Abstract*- In this work, we present a complete process to obtain a pure phase modulator based on a twisted nematic liquid crystal display (TNLCD). We first describe a simple technique for measuring the physical parameters of each cell of this device with no ambiguities, and we use them to build a simplified model of the display based on the Jones matrix theory. Then, after selecting a proper optical system for pure phase modulation, an optimal configuration of the system is searched using a numerical algorithm. This procedure drastically reduces the voluminous experimental stage this task normally requires. The presented method is verified through experimental results, finding an excellent agreement between the measurements and the predicted data.**

I. Introduction

Nowadays, the majority of liquid crystal opto-electronic applications are strongly related to imaging systems (LCD). However, over the last two decades, they are progressively being introduced in widely diverse areas, especially in the research field, due to their great versatility and low cost compared to other alternatives [1]. In many of these applications, spatial light modulators (SLMs) which only act on the two-dimensional phase front of a given beam are requested. Among others, it is worth mentioning the adaptive optical techniques that compensate the phase spatial distortion due to propagation in turbulent media, used in astronomical observation, wireless communications and microscopy, and more recently in the generation of orbital angular momentum (OAM) carrier beams for high density modulations.

One of the alternatives for obtaining pure phase SLM, which is used in this article, is based on the utilization of twisted nematic liquid crystal displays (TNLCD). These devices consist of a two-dimensional array of liquid crystal cells, whose constituent molecules are irregularly distributed but maintain the same orientation locally, and can be controlled by the action of an external electric field. A progressive angular variation of the molecular orientation is printed on the manufacturing process, which justifies its denomination. In Fig. 1., the described structure is illustrated.

Nematic liquid crystals behaves as an uniaxial birefringent anisotropic media, in which its optical axis coincides with his molecular orientation. It is demonstrated that, when the device is sufficiently thick, the aforementioned molecular variation induces a rotation in the same direction of the polarization plane of a light beam propagating through the device. In the case of an external field being applied, an additional inclination of the molecular axis over the direction of propagation takes place, which modifies the birefringence of the medium and consequently the induced rotation over the polarization plane of the incident beam.

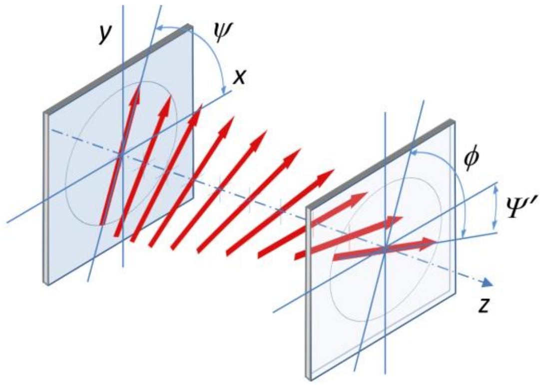

Fig. 1. Molecular orientation within a LC cell

It is well known that the combination of a TNLCD sandwiched between a polarizer and an analyser produces an amplitude modulation by the variation of the birefringence. That is the main application for TNLCDs, widely used in projection systems as an amplitude SLM. Nonetheless, with the same setting an additional phase modulation is produced.

This opens up a wide range of possibilities, including the attainment of a pure phase spatial modulator, in response to the aforementioned demand. In this sense, the work presented here proposes a complete method to configure and characterize a pure phase modulator based on a TNLCD, which also drastically reduces the experimental stage that is normally required.

II. Theory

For nematic liquid crystals, the values of the refractive index as a function of the orientation of the electric field are described by a revolution ellipsoid, as it is illustrated in Fig. 2. The major axis of the ellipsoid corresponds to the extraordinary axis, with refractive index n_{eo}, whereas the minor axis coincides with the ordinary axis, whose refractive index is n_o. According to that, a beam that propagates in the *z*-direction will experiment a refraction index n_{eo} when it is linearly polarized in the *x*-direction, while it will be n_o when it is polarized in the *y*-direction.

The aforementioned ellipsoid describes the macroscopic effect that the molecular distribution causes on the propagating beam, and his major axis coincides with the optical axis or molecular director of the medium. When an external field is applied, it causes the liquid crystal molecules to tilt over the propagation direction. In that case, an incident light beam that is linearly polarized in the *x*-direction will adjust to a new refraction index, $n_{eff}(\theta)$, that can be determined by the expression

$$\frac{1}{n_{eff}^2(\theta)} = \frac{\cos^2(\theta)}{n_o^2} + \frac{\sin^2(\theta)}{n_e^2}, \qquad (1)$$

which matches the equation of an ellipse in polar coordinates, whereas the refraction index associated to the *y*-direction will maintain the same value, n_o. The angle of rotation θ can be obtained as

$$\theta = \begin{cases} 0, & si\ V_{rms} \le V_c \\ \dfrac{\pi}{2} - 2\tan^{-1}\left(e^{-\frac{V_{rms}-V_c}{V_o}}\right), & si\ V_{rms} > V_c \end{cases} \qquad (2)$$

Where V_{rms} is the effective control voltage applied, and V_c is the threshold voltage, characteristic of the medium.

Considering the previous description, it is implicit that the birefringence, β, of a TNLC can be controlled electrically. This parameter is given by

$$\beta = (n_e - n_o)\frac{\pi l}{\lambda}, \qquad (3)$$

where l is the thickness of the liquid crystal layer and λ is the wavelength of the propagating beam.

As mentioned before, the molecules of a TNLCD follow a helix structure, which means that the molecular director rotates progressively in the direction of propagation. The angle of the molecular director at the input face, relative to the laboratory coordinate system, is called the *rubbing direction*, Ψ_D. The total variation of Ψ_D, from the input to the output is known as the *twist angle*, ϕ, and its typical values are $\pm 90°$. Both Ψ_D y ϕ remain constant when a voltage is applied to the cell.

In this work, we use a TNLCD where the voltage applied to each cell can be controlled by a specific software. That tool encodes the different voltages into a two-dimensional image in grey scale [2].

In order to develop a simple model to describe the optical behaviour of a TNLCD, the Jones matrix formalism is used. According to this method, the states of polarization of both input and outuput beam are represented by a 2x1 Jones vector, whereas an optical element is represented by a 2x2 Jones matrix. A complete optical system can be easily modelled by multiplying the Jones matrices of the constituent elements. Finally, the polarization state at the output of an optical system is found by taking the product of its Jones matrix and the input Jones vector.

In the most common model, the Jones matrix of a TNLC cell is found when it is consider as a sequence of N retarder waveplates, whose slow axes rotate progressively in the direction of propagation [3]. The final matrix is obtained when N→ ∞. In that case, the resulting expression will be

$$M_{TNLC} = e^{-j\beta} R(-\phi) \begin{pmatrix} \cos\gamma - j\frac{\beta}{\gamma}\sin\gamma & \frac{\phi}{\gamma}\sin\gamma \\ -\frac{\phi}{\gamma}\sin\gamma & \cos\gamma + j\frac{\beta}{\gamma}\sin\gamma \end{pmatrix} \qquad (4)$$

Where the parameter γ is defined as

$$\gamma = \sqrt{\phi^2 + \beta^2} \qquad (5)$$

Expression (9) will be used in the following sections for predicting the characteristics of a TNLCD as a spatial light modulator.

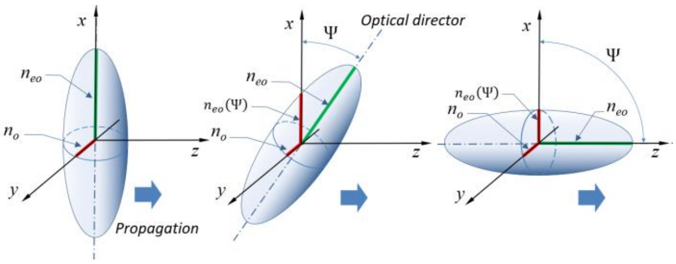

Fig. 2. Refraction indexes for different molecular orientations

III. Determination of the TNLCD Jones matrix

The main drawback of a commercial TNLCD is the lack of information provided by the manufacturer about the actual values of the physical parameters. This means that an experimental characterization of the device is frequently required. More specifically, in order to compose its Jones matrix, it is essential to measure the birefringence β, the *twist angle* ϕ and the *rubbing direction* Ψ_D.

Several methods have been proposed to determine these parameters. In this work, we use the Soutar and Lu [4] method as it provides measurements of ϕ, Ψ_D and β with a high level of accuracy.

For this method, the optical set-up consisted of a TNLCD sandwiched between a polarizer and an analiser, as illustrated in Fig. 3. The physical parameters of the cell can be derived by contrasting the measurement of the intensity transmitted through the system with its theorical value, when the polarizers are rotated simultaneously. A quarter-wave plate was added to the system to ensure uniform optical input power, since we used a linearly polarized source.

We obtained measurements for two particular settings of the angle of the first polarizer, ζ_1, and the analiser, ζ_2, known as *parallel configuration*, when $\zeta_1 = \zeta_2$, and a *crossed configuration* when $\zeta_1 = \zeta_2 + \pi/2$. The normalized intensity transmitted in *parallel configuration*, T_p, and *crossed configuration*, T_c, are given by

$$T_p = \left[\cos\gamma\cos\phi + \frac{\phi}{\gamma}\sin\gamma\sin\phi\right]^2 + \left[\frac{\beta}{\gamma}\sin\gamma\cos(2\zeta_1 - \phi - 2\psi_D)\right]^2 \qquad (6)$$

$$T_c = \left[-\cos\gamma\sin\phi + \frac{\phi}{\gamma}\sin\gamma\cos\phi\right]^2 + \left[\frac{\beta}{\gamma}\sin\gamma\sin(2\zeta_1 - \phi - 2\psi_D)\right]^2 \qquad (7)$$

It can be shown that Eqs. (6) and (7) are complementary, in the sense that

$$T_p + T_c = 1 \qquad (8)$$

Finally, the cell parameters β, ϕ and ψ_D can be determined with a nonlinear fitting of Eqs. (6) and (7) to the experimental curves T_p and T_c, respectively.

A. Ambiguities in the determination of the cell parameters

The last procedure does not provide an univocal determination of the TNLC parameters. The first source of ambiguity comes from the invariance of Eqs. (6) and (7) when the sign of the molecular twist, ϕ, changes, and also when Ψ_D is replaced by $\Psi_D + \pi/2$. The latter has the physical

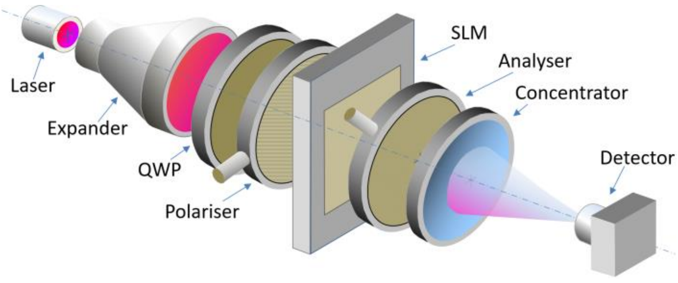

Fig. 3. Optical configuration for the measurement of the TNLCD parameters

meaning that the aforementioned method does not permit a distinction between the extraordinary and the ordinary axes

Several techniques have been proposed to overcome the indicated ambiguities. In this work, we used a specific polarimetry method based on the measurement of the normalized Stokes parameters. Those parameters can be expressed in terms of the physical parameters of the TNLC, as shown in the following equations

$$S_1 = (X^2 - Z^2)\cos[2(\phi + \zeta_1)] + 2XZ\sin[2(\phi + \zeta_1)]$$
$$+Y^2\cos[2(\phi + 2\Psi_D - \zeta_1)]$$
$$S_2 = (X^2 - Z^2)\sin[2(\phi + \zeta_1)] - 2XZ\cos[2(\phi + \zeta_1)]$$
$$+Y^2\sin[2(\phi + 2\Psi_D - \zeta_1)]$$
$$S_3 = -2Y(Z\cos[2(\Psi_D - \zeta_1)] + X\sin[2(\Psi_D - \zeta_1)]) \tag{9}$$

Where $X = \cos\gamma$, $Y = \frac{\beta}{\gamma}\sin\gamma$ and $Z = \frac{\phi}{\gamma}\sin\gamma$. In the laboratory, the parameter S_1 is obtained as the difference between the intensity transmitted at the output of an analiser when its transmission axis is oriented according along the x-axis and y-axis of the reference system. For the parameter S_2, the analiser must be oriented $\pm 45°$ with respect to the x-axis. Finally, to obtain the S_3, we calculate the difference of intensities for a right-handed circular polarization and a left-handed circular polarization.

The definition of the Stokes parameters as a function of the cell parameters eliminates all ambiguities of the Soutar and Lu method [5]. On the one hand, it is observed that replacing the value Ψ_D with $\Psi_\mathrm{D} + \pi/2$ changes the sign of S_3. On the other hand, a change of the sign of the parameter ϕ leads to different values of S_1, S_2 and S_3. The right values can be determined by comparing the measured curves with the different theorical alternatives.

B. *Experimental results*

At this point, the set-up presented in Fig. 3. was constructed, with the aim of determining the cell parameters of the LC-SLM LC2012 display. We used a 650 nm laser as a light source. The power transmitted in the parallel and cross configurations were measured, rotating the polarizers in 10-degree steps and setting the control voltage of the cell to zero. The results obtained were β_{Voff} = 2.2243 rad, ϕ = $\pm$1.5599 rad and Ψ_D = 0.8169 (+π/2) rad. The different ambiguities were solved using the method of the normalized Stokes parameters. For the measurement of S_3, an additional quarter-wave plate was added at the end of the set-up of Fig. 3, so it acts as a filter for circular polarization. Finally, we determined that the right values for the *twist angle* and the *rubbing direction* were ϕ = -1.5599 rad and Ψ_D = 0.8169 rad. A full characterization of the display requires an evaluation of

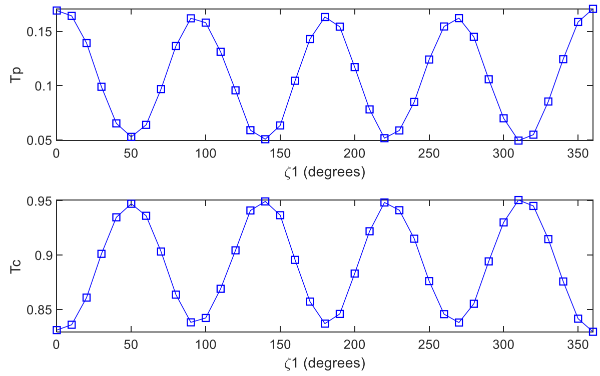

Fig. 4. Experimental curves obtained for the parallel configuration (upper curve) and crossed configuration (lower curve)

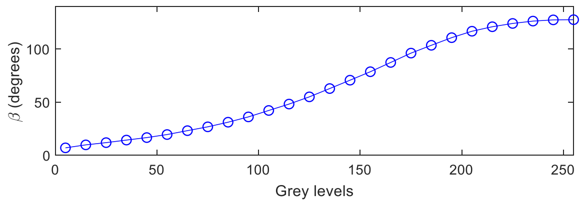

Fig. 5. Birefingence β as a function of the displayed grey level

the birefringence, β, as a function of the applied voltage. For that purpose, we repeated the first method varying the displayed grey level from 0 to 255. Since the values of ϕ and Ψ_D remain constant with the applied voltage, β can be directly extracted from Eqs. (6) and (7) using a nonlinear equation solving algorithm. The curves obtained after this procedure are shown in Fig. 4. and Fig. 5..

IV. Pure Phase Modulation

A. *Optical configuration for a pure phase modulation*

A change in the birefringence of the LC cell produces, in general, a coupled amplitude and phase modulation. However, it can be demonstrated that a nearly pure amplitude or phase modulation can be achieved when a configuration of optical elements is carefully chosen, so it sets the right polarization states of light at the input and the output of the SLM [6]. Normally, the search for these configurations requires a voluminous experimental procedure. Nevertheless, in this work, we proposed a numerical optimization method for this purpose based on the simplified model of Jones matrices, which drastically reduces the complexity of the process. The sufficiency of the model used, experimentally confirmed with excellent results, is motivated by the conjecture that the optimal configuration of the system does not depend on the complex edge effect in the TNLCD, which considerably complicates the corresponding model and whose effect is only noticeable at the limits of the voltage range, excluded for a pure phase modulation.

The proposed system consists of the polarizer – SLM – quarter-wave plate – analiser combination. For mathematical simplicity, the transmission angle of the first polarizer, ξ_1, is referred to the coordinate system in which the x-axis coincides with the optical director at the input surface, whereas the angle of the second polarizer, ξ_2, and the slow axis of the quarter-wave plate, η, are referred to the coordinate system where the x-axis coincides with the optical director at the output surface. The Jones vector at the output of this optical configuration can be written as

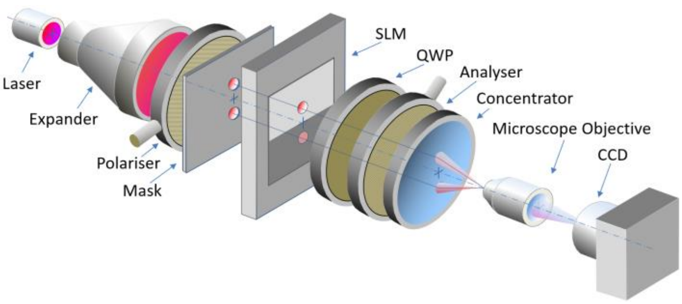

Fig. 6. Optical set-up for the measurement of the phase modulation

$$\begin{pmatrix} E_x^{out} \\ E_y^{out} \end{pmatrix} = P_H \cdot R(\phi + \xi_2) \cdot R(-\phi - \eta) \cdot W_{QWP} \cdot R(\phi + \eta) \cdot M_{TNLC} \cdot \begin{pmatrix} \cos \xi_1 \\ \sin \xi_1 \end{pmatrix} \tag{10}$$

where P_H represents the matrix of a linear polarizer with its transmission axis oriented along the x-axis, $R(\cdot)$ is the rotation matrix and W_{QWP} is the Jones matrix of a quarter-wave plate whose slow axis is oriented along the x-axis. Considering Eq. (11), it can be derived the expressions for the normalized intensity transmittance and the phase variation at the output of the system, given by

$$T = \frac{1}{2}\left[\cos\gamma \cos A + \frac{\sin\gamma}{\gamma}[\phi \sin A - \beta \cos C]\right]^2 + \frac{1}{2}\left[\cos\gamma \cos D - \frac{\sin\gamma}{\gamma}[\phi \sin D - \beta \cos B]\right]^2 \tag{11}$$

$$\psi = -\beta + \tan^{-1}\left(\frac{\cos \mathrm{A} + \frac{\tan\gamma}{\gamma}[\phi \sin \mathrm{A} - \beta \cos \mathrm{C}]}{-\cos \mathrm{D} + \frac{\tan\gamma}{\gamma}[\phi \sin \mathrm{D} - \beta \cos \mathrm{B}]}\right) \tag{12}$$

Where $A = \xi_1 - \xi_2$, $B = \xi_1 + \xi_2$, $C = 2\eta + \xi_1 - \xi_2$ and $D = 2\eta - \xi_1 - \xi_2$. The quest for pure phase modulation leads to a search of the values of ξ_1, ξ_2 and η_2 that minimize the variation of transmitted intensity, $\Delta T = T_{MAX} - T_{MIN}$, and maximize the range of the phase, ψ. That search was carried out through a numerical optimization procedure, with results $\xi_1 = 145°$, $\xi_2 = 174°$ y $\eta_2 = 9°$.

B. Phase and amplitude modulation measurement

In this stage, the phase modulation is measured experimentally using a double beam interferometer, as illustrated in Fig. 6. The TNLCD is illuminated with two collimated laser beams, generated by a double-hole mask. Each of these beams crosses a sector of the display that is set to a different voltage. One of the sectors has a constant grey level, whereas the other one variates throughout the range. A convergent lens behind the analiser acts as a concentrator, so an interference pattern emerges in its focal plane. That pattern is expanded by a microscope lens and finally recorded by a CCD camera. A voltage shift in the variable area of the SLM produces a phase modulation, and that causes a translation of the interference pattern. That effect is illustrated in Fig. 7.

The amplitude modulation can be evaluated by displaying a homogeneous image on the device and measuring the transmitted intensity for each grey level. The results obtained for amplitude and phase modulation are shown in Fig. 8. These results highlight the high degree of coincidence between the theorical model presented in this work and the real system.

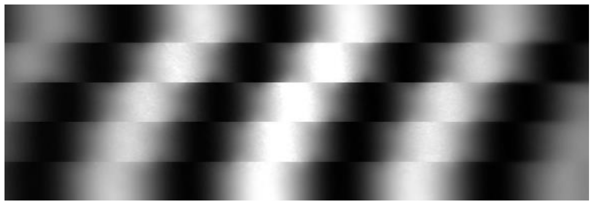

Fig. 7. Interference pattern shift due to a phase modulation

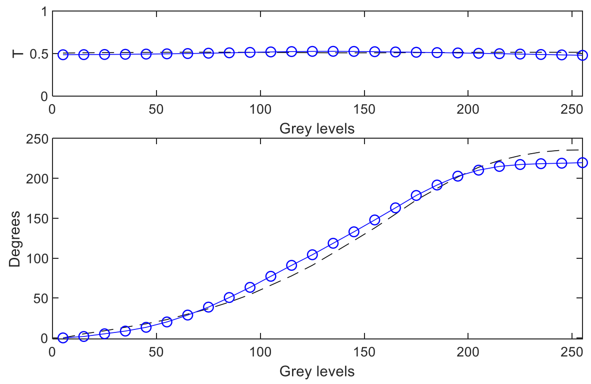

Fig. 8. Experimental (continuous line) and predicted (dashed line) curves for amplitude (upper curve) and phase (lower curve) modulation

V. Conclusions

In this work, we propose a complete method for configuring and characterizing a pure phase modulator based on a TNLC device, which drastically reduces the voluminous experimental stage that normally takes place. As a primary contribution, we present a numerical procedure for determining an optimal configuration of the proposed optical system by means of a simplified software model. Finally, an excellent degree of agreement between the predicted and the experimentally measured curves is demonstrated, which confirms the high potential of this method.

Acknowledgements

This work has been undertaken within the context of a Student Collaboration Grant in University Departments financed by the Spanish Ministry of Education and Vocational Training.

References

[1] J.M. Otón, et al. "Liquid-crystal phase only devices", *Journal of Molecular Liquids*, vol. 267, 11 2017.

[2] N.J. Gallego Molina, A. Puerta Notario, M. Castillo Vázquez, J.M. Garrido Balsells, "Simulador de dispositivos SLM basados en cristal líquido", in *XXXIII Simposium Nacional de la Unión Científica Internacional de Radio. URSI 2018*, Universidad de Granada, 4-7 de septiembre, (2018).

[3] V. Durán Bosch, "Optimización del funcionamiento de un modulador espacial de luz de cristal líquido mediante el modelo retardador-rotor. Aplicaciones en óptica adaptativa.", PhD Tesis, Universidad de Valencia, pp. 38-40, 2007.

[4] Soutar and K. Lu, "Determination of the physical properties of an arbitrary twisted-nematic liquid crystal cell," *Opt. Eng*., Vol. 33, No. 8, p. 2704, 1994.

[5] V. Durán, J. Lancis and E. Tajahuerce, "Cell parameter determination of a twisted-nematic liquid crystal display by single-wavelength polarimetry", *Journal of Applied Physics* 97, (2005)

[6] A. Marquez, C. C. Iemmi, I. S. Moreno, J. A. Davis, J. Campos and M. J. Yzuel, "Quantitative prediction of the modulation behaviour of twisted nematic liquid crystal displays based on simple physical model", *Optical Engineering* 40 (2001)