

# BEST RANK- $k$ APPROXIMATIONS FOR TENSORS: GENERALIZING ECKART-YOUNG

ALICIA TOCINO SÁNCHEZ

Joint work with Jan Draisma and Giorgio Ottaviani. Given a tensor  $f$  in a Euclidean tensor space, we are interested in the critical points of the distance function from  $f$  to the set of tensors of rank at most  $k$ , which we call the critical rank-at-most- $k$  tensors for  $f$ . When  $f$  is a matrix, the critical rank-one matrices for  $f$  correspond to the singular pairs of  $f$ . The critical rank-one tensors for  $f$  lie in a linear subspace  $H_f$ , the critical space of  $f$ . Our main result is that, for any  $k$ , the critical rank-at-most- $k$  tensors for a sufficiently general  $f$  also lie in the critical space  $H_f$ . This is the part of Eckart-Young Theorem that generalizes from matrices to tensors. Moreover, we show that when the tensor format satisfies the triangle inequalities, the critical space  $H_f$  is spanned by the complex critical rank-one tensors. Since  $f$  itself belongs to  $H_f$ , we deduce that also  $f$  itself is a linear combination of its critical rank-one tensors. For simplicity, we will focus on binary forms during the talk.

## REFERENCES

- [1] G. Ottaviani, A. Tocino, *Best rank  $k$  approximation for binary forms*, *Collectanea Mathematica*, **69**, 163-171 (2018).
- [2] J. Draisma, G. Ottaviani, A. Tocino, *Best rank- $k$  approximations for tensors: generalizing Eckart-Young*, *Research in the Mathematical Sciences*, **5:27**, 1-13 (2018).

DEPARTAMENTO DE ALGEBRA, GEOMETRÍA Y TOPOLOGÍA, UNIVERSIDAD DE MÁLAGA, AP. 59, 29080 MÁLAGA, SPAIN

*E-mail address:* `alicia.tocino@uma.es`