OPTION PRICING AND STOCHASTIC OPTIMIZATION

Nataliya Shchestyuk
National University "KyIv-Mohila Academy", Ukraine
n.shchestuk@ukma.edu.ua

Abstract

In this paper will be demonstrated that the link between optimal option value, risk measuring and risk managing is especially close, and it is given by stochastic optimization.

Post the financial crisis of 2008 it has been clear that risk considerations must enter into the valuation of derivatives, previously performed in an entirely "risk neutral world". The average value-at-risk is related to the problem of investor [1], which we use for option pricing. Let $Y = (S - K)^+$ is payoff for call option, where strike price K and premium C is known to investor and distribution of underlying risky assets S is modeled. Suppose that an investor has to decide about the amount X of invest before the actual available income is given to him by random variable The total piecewise linear profit/cost function is

$$H = X - u(Y - X)^- + l(Y - X)^+ = (1 - l)\left(X - \frac{(u - l)}{(1 - l)}(Y - X)^- \right) + lY .$$

So we have optimization problem and we can use the fact ((see [Rockafellar and Uryasev (2000)])) , that the average value-at-risk may be represented as the optimal value

$$AV@R_\alpha(Y) = \max\left\{ X - \frac{1}{\alpha}E\left((Y - X)^-\right); X \in R \right\}$$

and the maximum is attained. The optimal decision X should maximize the expected profit $E(H(X,Y))$. Finally the optimal decision (optimal option price) is $x = G^{-1}(\alpha)$. where $\alpha = \frac{1 - l}{u - l}$, G is the distribution function of Y.

Also we discuss Student-like model with dependence (FAT models) and determine optimal amount of invest X for this case. Meanwhile we measure and manage risk for the FAT model.

References

1. G. Ch Pflug, Romisch Werner. Modeling, Measuring and Managing Risk // World Scientific Publishing Co. Pte. Ltd. — 2007. P.220.

2. F. Castely, N. Leonenko, N. Shchestyuk. Student-like models for risky asset with dependence // Stochastic Analysis and Applications. – 2017**.** – Vol 35, № 3, pp 452-464