A New Tractable Model to Estimate the Uplink Inter-Cell Interference in LTE

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***Abstract*— This paper presents a tractable model to estimate the uplink Inter-Cell Interference (ICI) in fully loaded LTE networks. Interfering users are considered to be uniformly distributed in an annulus so that evaluation of the interference power is obtained asymptotically for a high number of users. Model validity is proved through Monte Carlo simulations, showing that analytical results are close to simulation results even for a low number of users per cell.**

I. INTRODUCTION

New generation cellular networks, like Long Term Evolution (LTE), are targeted to support aggressive frequency reuse patterns in order to maximize the spectral efficiency, though at the expense of increasing the Inter-Cell Interference (ICI).

In the uplink, interfering signals come from moving User Equipments (UEs) located anywhere in neighboring cells, so the analytical methods about the downlink interference cannot be directly applied to the uplink.

Currently, the available literature related to the estimation of the ICI in the uplink for LTE is rather limited. For instance, a method to assess the uplink ICI interference in a semi-analytical way is presented in [1]. In [2], an analytical method to compute the ICI in uplink when a Soft Frequency Reuse (SFR) scheme is presented, although the method only takes into account the interference generated from the six nearest surrounding cells. In general, previous works in this field presents either time-consuming system-level simulations or complex analytical expressions (corresponding to just one or few interfering cells) to compute statistical characteristics of ICI.

In this paper, we present a simple model to estimate the uplink ICI generated by a wide set of interfering cells by using a continuous approximation for interference origin.

The remainder of this paper is structured as follows. A description of the system model is given in Section II. In Section III we describe our proposed analytical uplink continuous interference model. The results both from our analytical model and Monte-Carlo simulations are analyzed in Section IV. Finally, some concluding remarks are given in Section V.

II. SYSTEM MODEL

Initially, we focus on the uplink of an Orthogonal Frequency Division Multiple Access (OFDMA) cellular system with M subcarriers and full frequency reuse as illustrated in Fig.1. We consider a system with an arbitrary number C of cells. The observed cell BS is located at (0,0) and equipped with one omnidirectional antenna. The remaining $C-1$ cells will be considered as interfering cells. There are a number of users whose positions are assumed to be i.i.d. random variables uniformly distributed in the system. Those UEs from neighboring cells that reuse the same time/frequency resources generate signals that interfere at the receiver BS under analysis. Actually, in the case of reuse 1, all UEs might contribute to the overall interference at the observed BS.

Received signals at the BS experience a path loss given as a power gain with the following expression $g(x,y) = \alpha \cdot (x^2+y^2)^{-\beta/2}$, being $r = \sqrt{(x^2+y^2)}$ the distance (in meters) between transmitter and receiver, whereas α and β are constants that depend on the specific environment (rural, urban, etc.) [3].

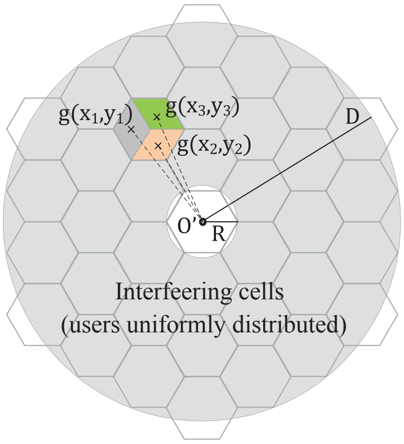

Fig. 1. Interfering cells in a scenario with frequency reuse 1

The following premises are assumed:

1) Interfering UEs are placed in an annulus. Annulus inner radius corresponds to the observed cell radius R whereas its outer radius D is associated with the number of cells in the scenario (see Fig. 1). The central area of radius R (within the observed cell) can be considered exempt from interferences as transmitting UEs are orthogonally multiplexed in time and/or frequency band.
2) The area of the c-th cell (A_c) is partitioned in non-overlapping regions among the K_c served UEs within that cell, being $\bar{K}$ the average number of users per cell. The k-th region is associated with the k-th UE and it is centered in the position of that UE (x_k, y_k) having an area $\Delta s = A_c/K_c[\mathrm{m}^2]$, equal for all UEs in the cell.

3) We consider a scheduler that distributes equally all resources (time/frequency/power) among UEs (Round Robin). Furthermore, UEs always have data to transmit (full buffer source model), i.e. cells are fully loaded.
4) The transmitted power P_m for each subcarrier is constant. That means that the power spectral density of the transmitted symbols is constant $S_p(f) = P_t/B$, being B[Hz] the system bandwidth, and $P_t = M \cdot P_m$ the total transmitted power within a cell. Hence, each UE of cell c transmits a power $P_{t,k} = P_t/K_c$ in a bandwidth $B_k = B/K_c$.

III. PROPOSED UPLINK CONTINUOUS INTERFERENCE MODEL

The interference $I[W]$ of the BS of interest, placed at the origin of coordinates (O'), is given by

$$I = \sum_{c \in \mathcal{I}} \sum_{k=1}^{K_c} \frac{P_t}{K_c} g(x_k, y_k) \qquad (1)$$

where $(P_t/K_c) \cdot g(x_k, y_k)$ represents the interference generated by the k-th UE of cell c whereas $\mathcal{I}$ represents the set of $C-1$ interfering cells.

Let us consider that the number of UEs and subcarriers in the system is high enough to model such a scenario using asymptotic analysis with $K_c \to \infty \ \forall c$. This yields to

$$I = \lim_{\Delta s \to 0} \sum_{c \in \mathcal{I}} \sum_{k=1}^{K_c} \frac{P_t}{K_c \Delta x \Delta y} g(x_k, y_k) \Delta x \Delta y \qquad (2)$$

Note that if $K_c \to \infty$ then $\Delta s \to 0$ since $A_c = \lim_{K_c \to \infty} K_c \Delta s$ and $\Delta s = \Delta x \Delta y$. As UEs are assumed to be uniformly distributed within the interfering region and the resources are equally distributed among UEs, then the transmitted power can be asymptotically viewed as a constant transmitted power spatial density ρ [W/m^2] defined as

$$\rho \doteq \lim_{K_c \to \infty} \frac{P_t}{K_c \Delta x \Delta y} = \frac{P_t}{A_c} \qquad (3)$$

Considering (2) as an expression of the Riemann integral of the transmitted power spatial density ρ multiplied by the path gain $g(x, y)$, and taking the limit as described in (3), the following expression is obtained for the asymptotic interference power at the origin:

$$I = \int\int_{\mathcal{D}} \rho \cdot g(x, y) dx dy = \int\int_{\mathcal{D}} \rho \cdot g(r) r dr d\theta \qquad (4)$$

where the dominion of integration $\mathcal{D}$ corresponds to the annulus shown in Fig.1. Note that we have applied a change into polar coordinates (r, θ).

We can also consider a frequency reuse factor in our continuous model. It is clear that a frequency reuse higher than 1 will affect the dominion of integration $\mathcal{D}$ as the interference only comes from specific cells that depend on the reuse factor N_r. Such a dominion is quite complex and provokes that the integral in (4) can not be easily calculated analytically. However, in order to give a tractable expression for the average interference power, we propose the following expression, which is an equality for $N_r = 1$:

$$I \simeq \int_{\theta=0}^{2\pi} \int_{r=D_r-R}^{D} \frac{\rho}{N_r} \cdot g(r) r dr d\theta \qquad (5)$$

where $D_r = R \cdot \sqrt{3N_r}$ is the co-channel distance. Solving the integral in (5), the following expression is obtained:

$$I \simeq \frac{2\pi\alpha\rho(D^{2-\beta} - (D_r - R)^{2-\beta})}{(2-\beta)N_r} \qquad \beta \neq 2 \qquad (6)$$

Since $\beta > 2$ in all propagation models, previous expression has an asymptote when the interfering area tends to infinite, which is given for hexagonal cells ($A_c = 3\sqrt{3}R^2/2$) by

$$\lim_{D \to +\infty} I \simeq \frac{4\pi\alpha P_t}{3^{(\beta+1)/2}(\beta-2)R^{\beta}N_r^{\beta/2}} \qquad \beta > 2 \qquad (7)$$

IV. SIMULATION RESULTS

Simulations (using the parameters listed in Table 1) have been carried out following the premises described within the system model and measuring the interference power I at the observed BS. This procedure has been repeated $1e5$ times in order to obtain an estimation of the interference power probability density function (pdf), shown in Fig.2.

In this section, the numerical results provided by our proposed model are validated with Monte Carlo simulations whose main simulation parameters are listed in Table 1.

Parameter	*Value*
Total number of users	[50, 100, 1000, 10000]
UE Transmit Power (P_t)	20 dBm
Cell radius (R)	100 m - 3 km
Outer annulus radius (D)	5 km
Pathloss model	$\alpha = 6.19e(-15)$, $\beta = 5.14$ (Urban)
Frequency reuse factor (N_r)	1 and 3

Our proposed model assumes a continuous distribution of the interference within the whole dominion of integration $\mathcal{D}$. As a consequence, the deterministic average interference power I obtained from (6) provides more accurate values (compared to simulation results) as a higher number of transmitting users are considered in the system, as shown in Fig.2. Note that simulations results for 50 users in the system correspond to an average number $\bar{K} \approx 1.9$ users per cell, which means that in practice there is high probability of having non-interfering cells within the dominion of integration $\mathcal{D}$, thus reducing the average interference power. For increasing values of $\bar{K}$, the estimated pdf tends to a deterministic value given by (6), which actually corresponds to $\bar{K} \to \infty$. Nevertheless, the difference between the average value of the estimated pdf and theoretical results is already below 1 dBm for $\bar{K} > 3$.

Fig.3 shows the simulated vs. theoretical average interference power I for different cell radius R and frequency reuse factors N_r. As shown in the figure, small cells increase considerably the interference power, which can be reduced with a looser reuse factor. According to simulation results,

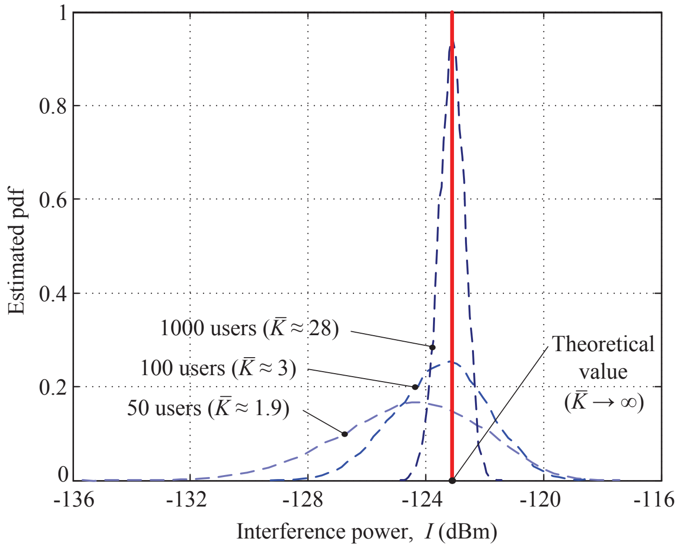

Fig. 2. Measured asymptotic interference power I for different number of users in the system ($N_r = 1$, $R = 1$ km, $D = 5$ km)

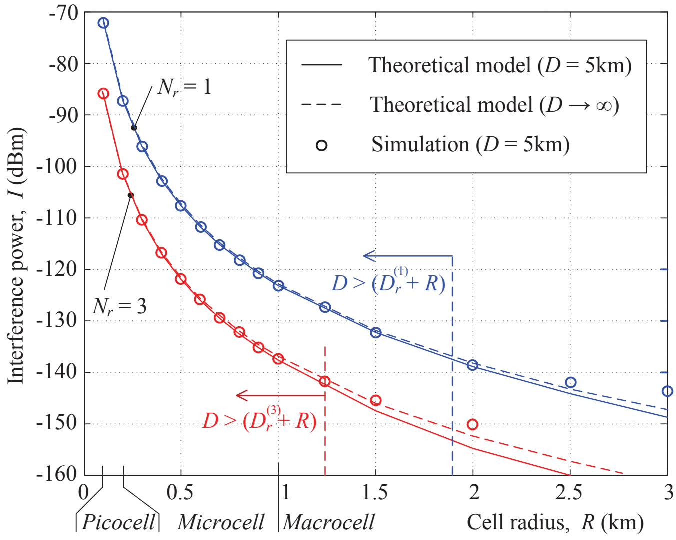

Fig. 3. Simulated (10000 users) vs. theoretical average interference power I for different cell radii and frequency reuse factors

our proposed model provides accurate estimations as long as $D > (D_r + R)$, i.e. the dominion of integration $\mathcal{D}$ includes, at least, the first sub-annulus of interfering cells. Note that for reuse 3, there is no interference in the simulations when $D > (D_r - R)$, i.e. $R > D/2$, as the number of interference cells within the dominion is null.

V. CONCLUSION

This paper presents a simple model to estimate the uplink ICI based on the fact that interfering users are uniformly spatially distributed in the system. A continuous model makes it possible to evaluate the asymptotic interference for a high number of users. Accurate estimations are obtained whenever the interfering area includes at least the first interfering sub-annulus, i.e. $D > (D_r + R)$. Monte Carlo simulation measures are within 1 dBm of analytical results even for an average number of users per cell $\bar{K} = 3$.

ACKNOWLEDGEMENTS

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