Using Educational Investment as a Screening and Signaling Device in the Labor Market

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Abstract

In the classical Spence´s model, workers have private information on their productivity and they use educational investment in order to signal their productive skills to an uninformed employer. Differently, we consider a two-period model in which an informed worker may invest in education in the first period, in which case she will not participate in the labour market and give up the market wage in that period, but her diploma will allow her to signal a greater productivity and receive a greater wage in the second period. As a result of the introduction of the investment period in the model, the educational investment may be used as a screening or a signalling device. We found that the relationship between the labour market conditions and the worker´s incentives to invest in education will depend on whether education plays a dominant role as a screening or a signalling device.

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1. Introduction

Economic theory started to analyze situations with asymmetric information in the seventies. In particular, the pioneer articles written by Spence (1973), Riley (1975) and Leland and Pyle (1977) suggested that those informed agents with the best qualities may use some observable actions in order to signal their private information in the market.

That literature on educational signaling usually assumes that the informed players incur the cost and obtain the profit from their actions in the same period. In particular, a worker invests in education in order to signal her productivity and she receives the wage in the labor market in the same period regardless of the level of education chosen.

Unlike those models, we consider a two-period signaling game in which a worker has private information on her productivity and decides whether she invests in education or not in the first period. If the worker does not invest in education, she will participate in the labor market in both periods, but if she invests in education, she will only work in the second period because she abandons the labor market during the investment period. We also introduce some uncertainty in this model. Specifically, the monetary value of the worker´s productivity will depend on the price at which the product can be sold in the market and the worker will have to choose their level of investment in the first period without knowing the price of the product in the second period.

This model will allow us to understand the effect of a change in the labor market conditions on the worker´s incentives to invest in education. In this setting, we obtain a separating equilibrium in which only a worker with an ability greater than a certain threshold invests in education, but this equilibrium may arise under two types of assumptions. First, when the cost of education decreases sufficiently with the worker´s ability, this equilibrium is obtained because the worker uses the educational investment as a signal of her productivity. Second, when the rate at which the cost of education decreases with the worker´s ability is sufficiently low, the separating equilibrium is also obtained, but now the reason why the worker with a low ability does not invest in education is the high opportunity cost of that investment.

In the scenario in which the cost of education decreases significantly with the worker´s ability, we obtain that the worker´s incentives to invest in education will become stronger when the worker is more patient, when the expected future price is greater or when the cost of education goes down. Interestingly, in the scenario in which the derivative of the cost of education with respect to the worker´s ability is sufficiently low, an increase in the worker´s level of patience, a rise in the expected future price and a lower cost of education will erode the incentives to invest in education. Finally, we found that these results are robust to the specification of the prior distribution of worker´s abilities and to the worker´s risk preferences.

This article is organized as follows. In the next section, we briefly describe the contribution of our model to previous literature. In the third section, we present the model. In the fourth, we obtain our main results with a risk-neutral worker and under the assumption of a uniform prior distribution of worker´s abilities, whereas we analyze the robustness of those results to different assumptions about the worker´s risk preferences and to a different specification of the prior distribution of abilities in the fifth and sixth sections, respectively. Finally, we summarize the main conclusions in the seventh section and the proofs of all lemmas and propositions are included in the appendix.

2. Literature Review

Since the seminal paper written by Spence (1973), theoretical models on the use of education as a signal of workers´ productivity in the labor market have proliferated. In most of those models, workers choose a level of education and receive the returns to their educational investment in the same period and consequently, they do not take into account the opportunity cost generated by the wages lost during the educational period. For this reason, previous literature on educational signaling cannot explain the interaction between the labor market outcomes and workers´ incentives to use their level of education as a signaling device.

Although previous researchers have studied some interactions between the labor market conditions and workers´ incentives to signal through education, they have focused on the relationship between the use of education as a signal and current employers´ decisions about retaining or promoting their employee to a better paid job when retention or promotion may signal workers´ productivity in the labor market (Waldman, 1984, 1990, 2016). These models assume that workers receive their wages in the same period regardless of the level of education they choose and consequently, they ignore the opportunity cost of education during the investment period.

Likewise, Kurlat and Scheuer (2021) assume that firms can directly observe imperfect information on workers´ types and the quality of that information is heterogeneous across firms. Due to the introduction of those assumptions, they found that signaling decreases if the cost is higher, if the demand for workers increases, or if firms´ expertise improves. These results are similar to those obtained in our model when the cost of education decreases sufficiently with the worker´s ability. However, the introduction of the opportunity cost of education allows us to obtain a different relationship between signaling and the labor market conditions when the relation between the cost of education and the worker´s ability is weaker. Additionally, in their model, education is a risky decision because equally productive and educated workers may receive different wages. The reason for this risk is that firms imperfectly evaluate workers´ productivities. In our model, the workers´ educational decision is risky because the conditions in the labor market after obtaining the diploma are uncertain.

3. Model

A worker has private information on her productivity, $t \in T = [t_0, t_n]$, where $0 < t_0 < t_n$. We assume that the prior distribution of this productivity is represented by a uniform distribution function and this is common knowledge. After observing her type, the worker chooses one of two possible levels of education: $e \in M = \{e_0, e_1\}$.

If the worker chooses e_0, it means that she does not study at the university and is employed in a company in periods 0 and 1. For simplicity, the cost of this level of education is assumed to be equal to zero. As this company competes a la Bertrand for hiring the worker, it will pay a wage equal to $P_0 E(t|e_0)$ to an uneducated worker in period 0, where $E(t|e_0)$ is the expected physical productivity among those workers who chose e_0 and P_0 is the price at which each unit of product can be sold in the market in period 0. In period 1, the uneducated worker will continue working in the labor market, in which case she receives a wage equal to the monetary value of her expected productivity: $P_1 E(t|e_0)$, where P_1 represents the price of the product in period 1. In order to avoid unnecessary complications, we assume that the future price of the product follows a simple process: $P_1 = \rho P_0 + \varepsilon$, where ρ represents the rate at which the price increases and ε is a random variable whose density function is $f(\cdot)$ and it represents fluctuations of the price around its long-run tendency. This density function is continuous and differentiable and is common knowledge. Moreover, the prior distribution of worker´s types and the distribution of ε are statistically independent. We assume that the support of ε is a closed interval: $[\overline{\varepsilon} - \Delta, \overline{\varepsilon} + \Delta]$. In order to avoid negative prices in period 1, we assume that $\Delta < \rho P_0 + \overline{\varepsilon}$. In this setting, the price per unit of product always increases due to some technological improvement, a rise in the quality of the product, the level of inflation in the market or any other reason, that is, we assume that $\rho > 1$. As $\rho > 0$, the prices of the product in both periods are positively correlated.

Finally, when the worker chooses to enroll on the university, e_1, she will incur a cost of that level of education in period 1: $c(t, e_1)$. As usual in models of educational signaling, we assume that the cost of education decreases with the worker´s type, that is, $\frac{\partial c(t,e_1)}{\partial t} < 0$. In this case, the worker will not work until completing the university degree in period 1 and the competitive company will offer her a wage equal to the monetary value of her productivity, which is $P_1E(t|e_1)$. As the price in period 1 is not known in period 0, the educational investment is risky.

Following standard notation, the discount factor will be represented by δ.

In this setting, we will obtain the perfect Bayesian equilibrium of the game, which satisfies the following conditions:

i. Each worker´s type will choose the level of education, e, that maximizes the discounted sum of utility levels: $u(t,e) = U[P_0E(t|e_0)1(e = e_0)] + \delta U[P_1E(t|e)]$, where $U(\cdot)$ is the worker´s utility function and $1(e = e_0)$ represents an indicator function, which is equal to one when the worker chooses the low level of education and zero otherwise.

ii. Given the level of education chosen by the worker, in each period the company will pay her a wage which is equal to the expected monetary value of productivity among those worker´s types with that level of education in equilibrium: $P_tE(t|e)$.

iii. The company´s beliefs must be consistent with the Bayesian rule in the equilibrium path. For example, when those worker´s types lower than t^* choose e_0 and those types greater than or equal to t^* choose e_1 in a separating equilibrium, the probability assigned by the company to type t after observing an educational level of e is given by $\mu(t|e) = \begin{cases} \frac{1}{t^*-t_0} & if\ e = e_0 \\ \frac{1}{t_n-t^*} & if\ e = e_1 \end{cases}$.

As shown by Mailath (1988), when the set of possible productivities of a worker forms a continuum, workers´ behavior in a separating equilibrium is completely determined because there is a unique separating equilibrium. In this article, we analyze the effects of a change in the labor market conditions on the separating equilibrium obtained.

4. Risk-Neutral Worker

In this section, we analyze the workers´ incentives to invest in education when they are risk-neutral.

The worker´s utility function, $U: \mathbb{R} \rightarrow \mathbb{R}$, is differentiable and strictly increasing, that is $U'(x) > 0\ \forall x \in \mathbb{R}$. In order to simplify the model, the worker´s utility from no money is normalized to be equal to zero, that is, $U(0) = 0$. Additionally, due to risk-neutrality[1], $U''(x) = 0\ \forall x \in \mathbb{R}$.

In this model, the level of education may reveal some information on the worker´s productivity in the labor market for two reasons. First, the educational investment has an opportunity cost which is equal to the wage lost during the investment period and as a result, only some worker´s types invest in education because the opportunity cost of education is lower than the additional wage received by educated workers. Therefore, education may serve as a screening mechanism in order to reveal partial information on the worker´s productivity. Second, as the cost of education decreases with the worker´s type, high-ability workers may invest in education in order to signal their higher productivity in the labor market. The dominant role of education as a screening mechanism or as a signaling device will depend on the rate at which the cost of education decreases with the worker´s type. In order to illustrate both cases, we consider two scenarios.

Screening. In this setting, we assume that the lowest type of worker will prefer the high to the low level of education when the employer identifies her type after observing the low level

[1] In the next section, we will analyse the robustness of our results to the specification of the worker´s risk-preferences.

and pays the ex-ante expected wage after observing the high. This assumption can be written as:

Assumption 1. $U(t_0 P_0) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_0(\rho P_0 + \varepsilon)] f(\varepsilon) d\varepsilon < \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon) - c(t_0, e_1)\right] f(\varepsilon) d\varepsilon.$

This assumption may be satisfied when the cost of education among the lowest types of worker is sufficiently low. Similarly, we assume that the highest type of worker prefers the low to the high level of education when the wage among non-educated workers is the monetary value of the ex-ante productivity and the wage among educated workers is equal to the value of the productivity of the highest type, that is:

Assumption 2. $U\left(\frac{t_0+t_n}{2} P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon)\right] f(\varepsilon) d\varepsilon > \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_n(\rho P_0 + \varepsilon) - c(t_n, e_1)] f(\varepsilon) d\varepsilon.$

This assumption may be satisfied when the cost of education among the highest-ability workers is sufficiently high.

In this screening scenario, there may be a pooling equilibrium in which all worker´s types choose the low level of education and another pooling equilibrium in which all types choose the high level. In this paper, we are only interested in a separating equilibrium in which only those worker´s types greater than a certain threshold invest in education and this equilibrium will arise when the following assumption is satisfied:

Assumption 3. $\left|\frac{\partial c(t, e_1)}{\partial t}\right| < \frac{P_0}{2\delta} \; \forall t \in [t_0, t_n].$

Signaling. In this scenario, we assume that the lowest type of worker will prefer the low to the high level of education when the employer identifies her type after observing the former and pays the ex-ante expected wage after observing the latter. This assumption can be written as:

Assumption 1´. $U(t_0 P_0) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_0(\rho P_0 + \varepsilon)] f(\varepsilon) d\varepsilon > \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon) - c(t_0, e_1)\right] f(\varepsilon) d\varepsilon.$

This assumption is satisfied when $c(t_0, e_1)$ is sufficiently high. Likewise, we assume that the highest type will prefer the high to the low level of education when the employer identifies her type after observing the high level of education and pays the monetary value of the ex-ante expected productivity after observing the low level, that is:

Assumption 2´. $U\left(\frac{t_0+t_n}{2} P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon < \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_n(\rho P_0 + \varepsilon) - c(t_n, e_1)] f(\varepsilon) d\varepsilon.$

This assumption holds providing that $c(t_n, e_1)$ is sufficiently low.

In this signaling scenario, there is no pooling equilibrium in which all worker´s types choose the high level of education. Although a pooling equilibrium in which all types choose the low educational level may arise, the out-of-equilibrium beliefs that support this equilibrium would be discarded by standard refinements, such as D1. In this scenario, the separating equilibrium will arise whenever the following assumption holds:

Assumption 3´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \frac{P_0}{2\delta}\ \forall t \in [t_0, t_n].$

In both scenarios, a separating equilibrium will arise. In that equilibrium, there will be a worker´s type, t^*, who will be indifferent between both levels of education, those types lower than t^* will choose e_0 and those types greater than t^* will choose e_1. In that equilibrium, the indifferent worker´s type, t^*, will satisfy the following equation:

$$U\left(\frac{t_0+t^*}{2}P_0\right)+\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right)f(\varepsilon)d\varepsilon=\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon)-c(t^*,e_1)\right]f(\varepsilon)d\varepsilon \quad (1)$$

The left-hand side of this equation shows the expected utility obtained by those workers´ types who choose to work in period 0, which is equal to the sum of the discounted levels of utility obtained by those types of worker, whereas the right-hand side shows the expected utility obtained by those worker´s types who complete the high level of education in period 1, which is equal to the discounted expected utility obtained in that period from the wage received minus the cost of education.

As a benchmark, we will analyze what happens when the cost of education does not change with the worker´s type, that is, let us assume that $c(t,e_1)=K\ \forall t\in T$. The equilibrium obtained in this setting allows us to isolate the effects of the introduction of the opportunity cost of education on the equilibrium obtained. Interestingly, the next lemma shows that this opportunity cost provides a mechanism in order to separate high from low-ability workers.

LEMMA 1. Under assumptions 1 and 2, when the cost of education does not change with t, there exists an equilibrium in which those types of worker lower than t^* choose e_0 and those types greater than t^* choose e_1, where $t^*\in T$.

As a result of the introduction of the opportunity cost in our model, this lemma shows that the employer may use the investment in education in order to distinguish low from high-productivity workers. Without the opportunity cost of education, it would be impossible to obtain this type of separating equilibrium when the cost of education is the same for all worker´s types. In particular, if the additional wage received by educated workers were greater than the cost of education, all worker´s types would choose the high level of education. Otherwise, no worker´s type would invest in education. Hence, separation would not arise. In equilibrium, lemma 1 shows that the cost of education, including the opportunity cost, is equal to the additional wage received by educated workers and for this reason, each type of worker is indifferent between investing and not investing in education.

LEMMA 2. Under assumptions 1 and 2, when the cost of education does not change with the workers´ type, t^* *increases with* δ *and decreases with* K.

In the separating equilibrium considered, the cost of education of the indifferent type of worker, including the opportunity cost, must be equal to the profit from the educational investment. For this reason, when the discounted profit from investing in education goes up as a result of a rise in the discount factor (δ) or a drop in the cost of education (K), the opportunity cost of education for the indifferent type of worker must also increase. As the opportunity cost of education is the wage received by non-educated workers, it will only increase when the indifferent type goes up so that the average productivity of non-educated workers goes up.

After the analysis of the benchmark with constant cost of education, we perform some comparative statics analysis in our general model. Under the assumptions of the screening and signaling scenarios, it is straightforward to see that there is a separating equilibrium like that described in lemma 1. In that equilibrium, the next proposition shows that the effect of a change in the parameters of the model on the worker´s incentives to invest in education will depend on the scenario considered.

PROPOSITION 1. In the screening scenario (under assumptions 1, 2 and 3), t^* *increases with* δ *and with* ρ *and decreases with* e_1. *In the signaling scenario (under assumptions 1´, 2´ and 3´),* t^* *decreases with* δ *and with* ρ *and increases with* e_1.

It is really easy to understand the intuition behind this proposition. Under the screening scenario, a greater discount rate, δ, or a greater expected price of the product in period 1, ρ, will increase the future profit from investing in education and the indifferent type´s opportunity cost of education should go up in order to compensate the rise in the expected future profit. For this reason, the indifferent worker´s type must increase so that the average productivity of those worker´s types who do not invest in education is higher. For similar reasons, when the monetary cost of education rises, that is, when e_1 goes up, the opportunity cost of education must go down to compensate it and consequently, the indifferent worker´s type decreases. However, in the signaling scenario, the role of education as a signaling device outweighs the effect of its opportunity cost. As a result, when δ or ρ goes up, the discounted profit from the educational investment increases and more low-ability workers invest in education (t^* goes down). Similarly, when the cost of education goes up (greater e_1), some workers with lower ability will not find it profitable to signal by investing in education (t^* goes up).

In the next proposition, we analyze the effect of a change in the price of the product in period 0 on the worker´s incentives to invest in education.

PROPOSITION 2. There exists a certain threshold, $K \in \mathbb{R}^+$*, such that* $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ *when* $\rho \lesseqgtr K$ *in the screening scenario (under assumptions 1, 2 and 3) and* $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ *when* $\rho \lesseqgtr K$ *in the signaling scenario (under assumptions 1´, 2´ and 3´).*

In this model, the effect of an improvement in the labor market today on the worker´s incentives to study will depend on the effect of that improvement on the expected prospects of the future labor market. In particular, when a rise in the current wage barely affects the wage expected by the worker in the future ($\rho < K$), a greater wage today will mainly imply a greater opportunity cost of studying at the university. On the contrary, when a greater wage today increases significantly the expected wage in the future ($\rho > K$), a greater wage today will mainly imply a greater expected profit from the educational investment. Then, in the signaling scenario, a rise in P_0 will lead to an increase in the opportunity cost of education and a lower proportion of educated types (greater t^*) when $\rho < K$ and to a greater expected profit from education and a greater proportion of educated types (lower t^*) when $\rho > K$. In the screening scenario, when $\rho < K$, the increase in the opportunity cost of education caused by a rise in P_0 has to be compensated with a reduction in that opportunity cost for the indifferent type, which gives rise to a lower proportion of educated types (lower t^*). Similarly, when $\rho > K$, the increase in the expected profit from education caused by a rise in P_0 has to be compensated with a greater opportunity cost of education for the indifferent type and for this reason, t^* increases.

5. Results with Different Risk Preferences

In this section, we obtain the equilibrium of the model when the worker is not risk-neutral. Now, the worker´s utility function is $U(x)$, which is continuous and twice differentiable, and we assume that $U'(x) > 0 \; \forall x \geq 0$. In order to avoid negative values of the utility function, we assume that $U(x) > 0 \; \forall x > 0$.

In this setting, we consider the usual measure of the worker´s risk-preferences:

$$\gamma_x = -\frac{U''(x)}{U'(x)}x \qquad (2)$$

In this section, we consider two situations. First, we study what happens when the worker is risk-averse, in which case $U''(x) < 0, \gamma_x > 0 \; \forall x > 0$. In this scenario, γ_x is the Arrow-Pratt measure of relative risk-aversion. Second, we obtain our results when the worker is risk-lover: $U''(x) > 0, \gamma_x < 0 \; \forall x > 0$. In this scenario, γ_x is a measure of the relative risk-loving.

As in the previous section, we consider the screening and signaling scenarios:

<u>Screening</u>. Once again, we consider assumptions 1 and 2 and substitute assumption 3 with the following condition:

Assumption 4. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \frac{P_0}{2\delta\overline{U}(t)} U'\left(\frac{t_0+t}{2} P_0\right) - \frac{1}{\overline{U}(t)} \int_{\overline{\varepsilon}-\Delta}^{\overline{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{U'\left[\frac{t+t_n}{2}(\rho P_0+\varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0+\varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon \; \forall t \in [t_0, t_n]$.

Where $\overline{U}(t) = \int_{\overline{\varepsilon}-\Delta}^{\overline{\varepsilon}+\Delta} U'\left[\frac{t+t_n}{2}(\rho P_0+\varepsilon) - c(t,e_1)\right] f(\varepsilon) d\varepsilon > 0$.

Signaling. In this set-up, we consider assumptions 1´ and 2´ and substitute assumption 3´ with the following condition:

Assumption 4´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \frac{P_0}{2\delta\overline{U}(t)} U'\left(\frac{t_0+t}{2} P_0\right) - \frac{1}{\overline{U}(t)} \int_{\overline{\varepsilon}-\Delta}^{\overline{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{U'\left[\frac{t+t_n}{2}(\rho P_0+\varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0+\varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon \; \forall t \in [t_0, t_n]$.

When the worker is risk-neutral, $\overline{U}(t) = \kappa = U'(x) \; \forall x$ and assumptions 4 and 4´ are equivalent to assumptions 3 and 3´ respectively.

Assuming that the prior distribution function of the worker´s productivity is uniform, $t \sim U[t_0, t_n]$, and that it is independent of the distribution of ε, once again, the worker´s type, t^*, who is indifferent between studying and not studying in a separating equilibrium will be given by equation (1).

The effects of changes in δ, e_1 and P_0 on t^* are the same as those described in propositions 1 and 2 in the screening and signaling scenarios.

Now, we analyze the effects of a parallel change in the measure of the worker´s risk-preferences, that is, we assume that $|\gamma_x^n| = |\gamma_x^o| + \alpha$, where γ_x^o and γ_x^n are the old and new measures of risk-preferences and $\alpha \in \mathbb{R}^+$ is the change in that measure, which is the same for all levels of consumption. Our next proposition shows the relationship between this type of change in the workers´ risk-preferences and their incentives to invest in education.

PROPOSITION 3. There exists $m \in \mathbb{R}^+$ *such that:*

i. *When either the worker is risk-averse and the assumptions of the screening scenario are satisfied or the worker is risk-lover and the assumptions of the signaling scenario are satisfied,* $\frac{\partial t^*}{\partial \alpha} \gtrless 0$ *if* $t_n - t_0 \lesseqgtr m$.

ii. *When either the worker is risk-averse and the assumptions of the signaling scenario are satisfied or the worker is risk-lover and the assumptions of the screening scenario are satisfied,* $\frac{\partial t^*}{\partial \alpha} \lesseqgtr 0$ *if* $t_n - t_0 \lesseqgtr m$.

In our model, the educational investment plays two roles. First, it is a mechanism used by the employer to separate low from high-productivity workers. This screening role arises because the opportunity cost of education increases with t^* and when t^* is sufficiently high, low-productivity workers prefer not to incur that cost. Second, the investment in education is a signaling device used by the worker. This signaling role arises because the monetary cost of education incurred by the indifferent type decreases with t^*. As a result, high-productivity workers signal their low cost of education by achieving the high educational level. This analysis shows that the screening role of education is strengthened when the cost of education increases with t^*, whereas the signaling role is reinforced otherwise.

In this setting, workers have to decide whether they invest in education or not in period 0 with perfect knowledge of the price in that period, but they do not know the price of the product in period 1 and for this reason, the wage received in that period is uncertain. Under these assumptions, the wage received by the worker in period 1 is the monetary value of the expected productivity:

$$w_1(e_i) = E(t|e_i)(\rho P_0 + \varepsilon) \ \forall i \in \{0,1\} \quad (3)$$

Where $E(t|e_i)$ is the expected productivity of those worker´s types who chose e_i in period 0. This wage is the sum of a certain ($E(t|e_i)\rho P_0$) and an uncertain component ($E(t|e_i)\varepsilon$).

In the separating equilibrium considered, the educated worker takes greater risks than the uneducated because $\left|\frac{t_0+t^*}{2}\varepsilon\right| > \left|\frac{t^*+t_n}{2}\varepsilon\right|$ $\forall\varepsilon$. Then, the additional risk assumed by the educated worker ($\left|\frac{t_n-t_0}{2}\varepsilon\right|$) is proportional to $t_n - t_o$.

Additionally, the certain components of the wage received by the educated ($\frac{t^*+t_n}{2}\rho P_0$) and uneducated workers ($\frac{t_0+t^*}{2}\rho P_0$) in period 1 are different. For this reason, the risk premium demanded by an educated worker will differ from that demanded by an uneducated worker when that employee is risk-averse. Similarly, the price the educated and uneducated workers would be willing to pay in order to take additional risk will be different when the worker is risk lover.

Now, it is easy to understand the results shown in proposition 3. We start with the analysis of the risk-averse worker. We distinguish two cases. First, when $t_n - t_o$ is sufficiently high, an increase in t^* will make the additional risk associated with the educational investment much greater and as a result, the cost of education will increase among risk-averse workers. As shown above, this direct relationship between the cost of education and the indifferent type of worker strengthen the screening role of education. As described by part i of proposition 3, in the screening scenario, when the degree of risk-aversion goes up, the cost of the educational risk increases and t^* decreases in order to compensate that additional cost by reducing the opportunity cost of education. However, in the signaling scenario, an increase in the worker´s risk aversion will weaken the signaling role of education and for this reason, t^* increases and the proportion of worker´s types who invest in education to signal their productivity decreases (see part ii of proposition 3). Second, when $t_n - t_o$ is sufficiently low, the expected productivities among educated and uneducated workers will be similar and the wages received will barely increase with education. In this case, the highest worker´s types will invest in education as long as the risk premium associated to the educational investment is lower than that associated to being uneducated in period 1. Hence, when t^* goes up, the risk premium associated to education is even lower and this relationship weakens the screening role and reinforces the signaling role. As shown by part i of proposition 3, in the screening scenario, when the worker´s risk aversion goes up, the screening role is even more weakened, which implies that t^* increases and the proportion of worker´s types who invest in education goes down. In the signaling scenario, a rise in the worker´s risk aversion reinforces the signaling role of education even more and for this reason, t^* decreases and the proportion of worker´s types who invest in education goes up as shown by part ii of proposition 3.

To finish off the analysis of proposition 3, we describe the workers´ incentives when they are risk-lover. Once again, when $t_n - t_o$ is sufficiently high, the risk associated with the educational investment increases significantly with t^*, which reduces the net cost of education incurred by the risk-lover worker. Consequently, the screening role of education is weakened, whereas the signaling role is reinforced. In the screening scenario, when the worker´s preferences for risk are stronger, the screening role of education is more weakened and for this reason, part ii of proposition 3 shows that t^* goes up and the proportion of worker´s types who choose the high level of education goes down. In the signaling scenario, an increase in the measure of the worker´s risk loving will reinforce the signaling role of education even more and as a result, t^* goes down and more worker´s types will invest in education as shown by part i of proposition 3. Finally, when $t_n - t_o$ is sufficiently low, the uneducated and educated workers receive similar wages and both types of worker take similar risk. Therefore, the only reason why the highest types incur the cost of education must be that the measure of risk loving decreases with the expected income. When t^* goes up, the monetary cost of education incurred by the indifferent type goes down and that type´s expected income goes up, which implies that the measure of risk loving and the expected utility from investing in education go down. As a result, the screening role of education is reinforced and the signaling role is weakened when $t_n - t_o$ is sufficiently low and the effects of a change in α on t^* are reversed as shown by proposition 3.

In this section, we have shown the relationship between the workers´ preferences for risk and their incentives to invest in education in our model. Regardless of whether the worker is risk-averse or risk-lover, the effect of a change in other parameters on the separating equilibrium obtained are the same as those described by propositions 1 and 2. As shown by propositions 1, 2 and 3, the comparative statics considered crucially depends on whether the assumptions of the screening or signaling scenario are satisfied. For this reason, we add two lemmas in order to determine some parametric conditions under which those settings will arise.

LEMMA 3. If the worker is risk-averse, there exist $m_1, \overline{\gamma}_1, \overline{\gamma}_2, \Delta_1, \Delta_2 \in \mathbb{R}^+$ *such that the assumptions of the screening scenario will be satisfied as long as* $t_n - t_0 > m_1, \gamma_c > \overline{\gamma}_1 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1 \; \forall t \in [t_0, t_n]$, *whereas the assumptions of the signaling scenario will hold as long as* $t_n - t_0 < m_1, \gamma_c > \overline{\gamma}_2 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_0, t_n]$.

LEMMA 4. If the worker is risk-lover, there exist $m_2, \overline{\gamma}_3, \overline{\gamma}_4, \Delta_3, \Delta_4 \in \mathbb{R}^+$ *such that the assumptions of the screening scenario will be satisfied providing that* $t_n - t_0 < m_2, \gamma_c > \overline{\gamma}_3 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3 \; \forall t \in [t_0, t_n]$, *whereas the assumptions of the signaling scenario will hold as long as* $t_n - t_0 > m_2, \gamma_c > \overline{\gamma}_4 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4 \; \forall t \in [t_0, t_n]$.

Like in the previous section with risk-neutrality, those lemmas show that the screening (signaling) scenario is more likely to appear when rate at which the cost of education decreases with the worker´s ability is sufficiently low (high). As shown in our analysis of proposition 3, when the worker is risk-averse, the risk associated with the educational investment strengthens the screening (signaling) role of education when $t_n - t_0$ is sufficiently high (low). For this reason, when the worker´s measure of risk aversion is sufficiently high, lemma 3 shows that the screening (signaling) scenario is more likely to appear if $t_n - t_0$ is sufficiently high (low). Finally, when the worker is risk-lover, our analysis of proposition 3 shows that the risk associated with education strengthens the signaling (screening) role of education when $t_n - t_0$ is sufficiently high (low). As a result, when the worker is sufficiently risk-lover, lemma 4 shows that the signaling (screening) is more likely to arise if $t_n - t_0$ is sufficiently high (low).

Our results suggest that the effect of changes in the labor market conditions on workers´ incentives to invest in education will depend on the dominant role of education as a screening or a signaling device. Specifically, lemmas 3 and 4 will guide future empirical researchers in order to determine the conditions under which a specific change may affect educational attainment in some way or another.

6. Robustness to the Specification of the Prior Distribution of Worker´s Types

In previous sections, we assumed that the worker´s type was drawn from a uniform distribution. In this section, we analyze the effect of specifying other types of distribution on the results obtained. In order to see the extent to which our results can be generalized, we will use the general relationship between the sender´s incentives to signal and the prior distribution of sender´s types obtained by Adriani and Sonderegger (2019) and Jewit (2004).

Now, let $f: T \rightarrow [0,1]$ denote the prior density function from which the sender´s type is drawn at the beginning of the signaling game we are describing. In our model, the signal can only take two values and the sender´s incentives to choose the high value is given by the following expression in the separating equilibrium considered:

$$\phi(t^*) = E(t|t > t^*) - E(t|t < t^*) \quad (4)$$

Where t^* is the worker´s type who is indifferent between the low and high values of the signal. In this setting, $\phi(t^*) > 0$ measures the worker´s incentive to invest in education.

We summarize the results obtained by Adriani and Sonderegger (2019) and Jewitt (2004) in the next lemma.

LEMMA 5. $\phi'(t^*)$ *will depend on the shape of* $f(t)$ *as follows:*

I. *If* f *is an everywhere increasing (decreasing) function, then* ϕ *will be decreasing (increasing) everywhere.*

II. *If* f *is strictly increasing and then decreasing (unimodal), then there exists* $t_m \in [t_0, t_n]$ *such that* ϕ *is strictly decreasing when* $t < t_m$ *and strictly increasing when* $t > t_m$*. Moreover, if* f *is symmetric, then* t_m *coincides with the mode of* f.

III. *If* f *is strictly decreasing and then increasing, then there exists* $t_M \in [t_0, t_n]$ *such that* ϕ *is strictly increasing when* $t < t_M$ *and strictly decreasing when* $t > t_M$*. Moreover, if* f *is symmetric, then* t_M *coincides with the anty-mode of* f.

The proof of this lemma can be found in Adriani and Sonderegger[2] (2019).

In this setting, when the worker is risk-neutral, the worker´s type who is indifferent between investing or not investing in education, t^*, will be given by:

$$U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] = \delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \quad (5)$$

Once again, we consider both scenarios previously described.

<u>Screening</u>. We assume that the following conditions will be satisfied:

Assumption 5. $U(t_0 P_0) + \delta U\big[t_0\big(\rho P_0 + E(\varepsilon)\big)\big] < \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big) - c(t_0, e_1)\big]$.

[2] See their lemmas 1 and 2.

Assumption 6. $U[E(t)P_0] + \delta U[E(t)(\rho P_0 + E(\varepsilon))] > \delta U[t_n(\rho P_0 + E(\varepsilon)) - c(t_n, e_1)]$.

Assumption 7. $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| < \frac{P_0}{\delta}\frac{\partial E(t|t < t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}(\rho P_0 + E(\varepsilon))\ \forall t^* \in [t_0, t_n]$.

Signaling. We assume that the following conditions will be satisfied:

Assumption 8. $U(t_0 P_0) + \delta U[t_0(\rho P_0 + E(\varepsilon))] > \delta U[E(t)(\rho P_0 + E(\varepsilon)) - c(t_0, e_1)]$.

Assumption 9. $U[E(t)P_0] + \delta U[E(t)(\rho P_0 + E(\varepsilon))] < \delta U[t_n(\rho P_0 + E(\varepsilon)) - c(t_n, e_1)]$.

Assumption 10. $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \frac{P_0}{\delta}\frac{\partial E(t|t < t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}(\rho P_0 + E(\varepsilon))\ \forall t^* \in [t_0, t_n]$.

As in the previous section, we also assume that the worker´s utility from the minimum level of consumption in period 1 is positive, that is, $U(t_0 P_0) > 0$.

Now, we can determine the effect of a change in the parameters of the model on the worker´s incentives to invest in education.

PROPOSITION 4. For any prior distribution of sender´s types, t^* *increases with* δ *and decreases with* e_1 *in the screening scenario and* t^* *decreases with* δ *and increases with* e_1 *in the signaling setting. Furthermore, there exists a certain threshold,* $K \in \mathbb{R}^+$, *such that* t^* *decreases (increases) with* P_0 *when* $\rho < K$ $(\rho > K)$ *in the screening set-up, but* t^* *increases (decreases) with* P_0 *when* $\rho < K$ $(\rho > K)$ *in the signaling scenario.*

These results are the same as those included in propositions 1 and 2, which means that the relationship between the worker´s incentives to invest in education and the labor market outcomes is robust to the specification of the prior distribution of sender´s types when the worker is risk-neutral.

When the prior distribution of worker´s types is concentrated on the highest types or when $f(\cdot)$ is increasing ($\phi(\cdot)$ decreases everywhere as shown in lemma 5), then the screening scenario is more likely to appear. On the contrary, when the prior distribution of worker´s types is concentrated on the lowest types or when $f(\cdot)$ is decreasing ($\phi(\cdot)$ increases everywhere as shown in lemma 5), then the signaling scenario is more likely to arise. Therefore, some sufficient conditions for each scenario are shown in our next lemma.

LEMMA 6. There exists $c_1 \in \mathbb{R}^+$ *such that the screening scenario will arise providing that* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1\ \forall t \in [t_0, t_n]$ *and* $f(\cdot)$ *is increasing everywhere. Likewise, there exists* $c_2 \in \mathbb{R}^+$ *such that the signaling scenario will arise as long as* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2\ \forall t \in [t_0, t_n]$ *and* $f(\cdot)$ *is decreasing everywhere.*

This lemma shows that the screening scenario will be more likely to arise in those labor markets where most workers are highly productive and the educational system is universal. On the contrary, high ability workers will be able to use education as a signal of their productivity as long as most workers are less productive and the educational system is more selective.

7. Conclusions

In this article, we analyze the effects of a change in the labor market conditions on people´s incentives to invest in education. In order to meet this goal, we extend Spence´s model of signaling by introducing the opportunity cost of education caused by the wage lost during the investment period. Specifically, we consider a two-period game in which a worker with private information on her productivity in the labor market has to decide whether she invests in education or not in period 1. Unlike the classical model, we assume that the worker cannot participate in the labor market during the investment period, but she may use a high level of education as a signal of her higher productivity in the second period, in which case she will receive a greater wage in period 2.

In this setting, the wage lost during the investment period is a key component of the opportunity cost of education in our model. Specifically, an equilibrium may arise in which only those workers with the highest ability invest in education. In this equilibrium, the wage received by uneducated workers is sufficiently high because some high types of worker do not invest in education. As a result, the wage lost by educated workers during the investment period is so high that low ability workers do not have incentives to choose the high level of education. Consequently, the employer will be able to identify those workers with the highest productivity by using the educational credentials. In contrast to the classical model, we found that this screening equilibrium may arise even when the monetary cost of education does not change with the worker´s ability, that is when the single-crossing condition is not satisfied.

On the contrary, when the single-crossing condition is satisfied and the cost of education decreases sufficiently with the worker´s ability, the classical signaling equilibrium will arise and high-ability workers will use the high level of education as a signal of their productivity. As expected, when the worker is more patient, when prospects of the labor market conditions in the future improve, or when the cost of education goes down, the incentives to invest in education become stronger under the conditions that lead to the signaling equilibrium. Interestingly, the opposite occurs under the conditions that lead to the screening equilibrium.

Therefore, our model shows the interaction between labor market conditions and the incentives to use education as a signaling device and we found that the results are robust to the specification of the workers´ preferences and to the prior distribution of workers´ types considered.

Appendix

Proof of Lemma 1. To start with, we define a function representing the worker´s additional profit from choosing e_0 when those types lower than $t \in T$ choose e_0 and those greater than t choose e_1:

$$F(t,\delta,K) = U\left(\frac{t_0+t}{2}P_0\right) + \delta\left[\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - K\right) f(\varepsilon)d\varepsilon\right] \quad \text{(A.1)}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \to \mathbb{R}$. Under assumptions 1 and 2, δ and K must take values such that $F(t_0, \delta, K) < 0$ and $F(t_n, \delta, K) > 0$. Since $\frac{\partial F}{\partial t} = \frac{P_0}{2} U'\left(\frac{t_0+t}{2}P_0\right) > 0$, there exists a unique $t^* \in [t_0, t_n]$ such that $F(t^*) = 0$, which is equation (1) defining the indifferent worker´s type in a separating equilibrium. QED.

Proof of Lemma 2. Given the function defined in (A.1), the indifference equation shown in (1) is equivalent to $F(t^*, \delta, K) = 0$. Then, we obtain the following derivatives:

$$\frac{\partial F(t^*,\delta,K)}{\partial \delta} = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - K\right) f(\varepsilon)d\varepsilon \quad \text{(A.2)}$$

Since $U(0) = 0$, $U'(\cdot) > 0$ and $\frac{t_0+t}{2}P_0 > 0\ \forall t \in [t_0, t_n]$, then $U\left(\frac{t_0+t^*}{2}P_0\right) > 0$. As a result, $\frac{\partial F(t^*,\delta,K)}{\partial \delta} < 0$ so that the indifference equation (1) is satisfied.

Similarly, it is straightforward to see that $\frac{\partial F(t^*,\delta,K)}{\partial K} > 0$. Finally, since $\frac{\partial F(t^*,\delta,K)}{\partial t^*} = \frac{P_0}{2} U'\left(\frac{t_0+t^*}{2}P_0\right) > 0$, we use the implicit function theorem in order to obtain the desired results:

$$\frac{\partial t^*}{\partial \delta} = -\frac{\frac{\partial F(t^*,\delta,K)}{\partial \delta}}{\frac{\partial F(t^*,\delta,K)}{\partial t^*}} > 0 \quad \text{(A.3)}$$

$$\frac{\partial t^*}{\partial K} = -\frac{\frac{\partial F(t^*,\delta,K)}{\partial K}}{\frac{\partial F(t^*,\delta,K)}{\partial t^*}} < 0 \quad \text{(A.4)}$$

This completes the proof of lemma 2. QED.

Proof of Proposition 1. First of all, we rewrite equation (1) as the following function of the indifferent worker´s type, t^*:

$$F(t^*, \delta, P_0, \rho, e_1) = U\left(\frac{t_0+t^*}{2} P_0\right) + \delta\left\{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \right.$$

$$\left. \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon) d\varepsilon\right\} = 0 \quad \text{(A.5)}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \times (1, +\infty) \times \mathbb{N} \rightarrow \mathbb{R}$ is the real function defined in equation (A.5). If we derive this function with respect to each variable, we obtain the following expressions:

$$\frac{\partial F}{\partial t^*} = \kappa\left[\frac{P_0}{2} + \delta c_1(t^*, e_1)\right] \quad \text{(A.6)}$$

$$\frac{\partial F}{\partial \delta} = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon) d\varepsilon$$

(A.7)

$$\frac{\partial F}{\partial P_0} = \kappa\left(\frac{t_0+t^*}{2} - \delta\frac{t_n - t_0}{2}\rho\right) \quad \text{(A.8)}$$

$$\frac{\partial F}{\partial \rho} = -\delta\kappa P_0 \frac{t_n - t_0}{2} \quad \text{(A.9)}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*, e_1) \quad \text{(A.10)}$$

Where $\kappa = U'(x)\ \forall x \in \mathbb{R}$, $c_1(t^*, e_1) = \frac{\partial c(t^*, e_1)}{\partial t^*}$ and $c_2(t^*, e_1) = \frac{\partial c(t^*, e_1)}{\partial e_1}$.

It is easy to see that $\frac{\partial F}{\partial t^*} > 0$ under assumption 3 and $\frac{\partial F}{\partial t^*} < 0$ under assumption 3´.

Additionally, since $U(0) = 0$, $U'(\cdot) > 0$ and $\frac{t_0+t^*}{2} P_0 > 0\ \forall t^* \in [t_0, t_n]$, then $U\left(\frac{t_0+t^*}{2} P_0\right) > 0$. As a result, $\frac{\partial F}{\partial \delta} < 0$ so that equation (A.5) is satisfied.

Finally, as shown by equations (A.9) and (A.10), $\frac{\partial F}{\partial \rho} < 0$ and $\frac{\partial F}{\partial e_1} > 0$. Thus, if we use the implicit function theorem, we obtain the desired results under assumption 3:

$$\frac{\partial t^*}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial t^*}} > 0 \qquad \text{(A.11)}$$

$$\frac{\partial t^*}{\partial \rho} = -\frac{\frac{\partial F}{\partial \rho}}{\frac{\partial F}{\partial t^*}} > 0 \qquad \text{(A.12)}$$

$$\frac{\partial t^*}{\partial e_1} = -\frac{\frac{\partial F}{\partial e_1}}{\frac{\partial F}{\partial t^*}} < 0 \qquad \text{(A.13)}$$

Under assumption 3´, these inequalities are reversed because $\frac{\partial F}{\partial t^*} < 0$ and this completes the proof of proposition 1. QED.

Proof of Proposition 2. From equation (A.8), we see that $\frac{\partial F}{\partial P_0} > 0$ when $\rho = 0$ and $\frac{\partial F}{\partial P_0}$ decreases with ρ. Then, there will be a threshold, $K \in \mathbb{R}^+$, such that $\frac{\partial F}{\partial P_0} > 0$ when $\rho < K$, whereas $\frac{\partial F}{\partial P_0} < 0$ when $\rho > K$. Once again, we use the implicit function theorem in order to obtain the desired results. In particular, under assumptions 1, 2 and 3, we conclude that $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho > K$. Likewise, under assumptions 1´, 2´ and 3´, we obtain that $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho > K$. QED.

Proof of Proposition 3. First of all, we rewrite equation (1) of the indifferent worker´s type, t^*:

$$F[t^*, \delta, P_0, \rho, e_1] = U\left(\frac{t_0+t^*}{2}P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left\{U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon = 0 \qquad \text{(A.14)}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \times [0, +\infty) \times \mathbb{N} \to \mathbb{R}$.

Next, we obtain the following derivative:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left\{\frac{\rho P_0+\varepsilon}{2} U'\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) - \left(\frac{\rho P_0+\varepsilon}{2} - \frac{\partial c(t^*,e_1)}{\partial t^*}\right) U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right]\right\} f(\varepsilon)\, d\varepsilon \quad \text{(A.15)}$$

In order to find out the sign of $\frac{\partial F}{\partial t^*}$, we rewrite the derivative shown in (A.15) as:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) - \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] - U'\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] f(\varepsilon)\, d\varepsilon$$

(A.16)

Using the mean value theorem (Lagrange theorem), we know that there exists $c_\varepsilon^* \in \mathbb{R}$ such that equation (A.16) can be expressed as[3]:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) - \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] U''(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] f(\varepsilon)\, d\varepsilon \quad \text{(A.17)}$$

Where c_ε^* may be different for each value of ε. Lastly, we substitute the second derivative of the utility function in the above expression with our measure of the worker´s relative risk-preference and obtain the following result:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] f(\varepsilon)\, d\varepsilon \quad \text{(A.18)}$$

Where $\gamma_{c_\varepsilon^*} = -\frac{U''(c_\varepsilon^*)c_\varepsilon^*}{U'(c_\varepsilon^*)}$. Therefore, $\frac{\partial F}{\partial t^*}$ is the sum of three components:

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[3] If $\frac{t_0+t^*}{2}(\rho P_0+\varepsilon) < \frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)$, then $c_\varepsilon^* \in \left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon), \frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right)$. Otherwise, $c_\varepsilon^* \in \left(\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1), \frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right)$.

i. $A = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) > 0$ because the worker´s utility function is strictly increasing.

ii. $B = \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0+\varepsilon) - c(t^*, e_1)\right] \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon.$

iii. $C = \delta \frac{\partial c(t^*,e_1)}{\partial t^*} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*, e_1)\right] f(\varepsilon)\, d\varepsilon.$ This component is a negative number because $\frac{\partial c(t^*,e_1)}{\partial t^*} < 0$ and the utility function is strictly increasing.

In the separating equilibrium considered, $\frac{\partial F}{\partial t^*} = 0$.

In order to determine the sign of B, we rewrite B in the following way:

$$B = \delta \left[\frac{t_n-t_0}{4} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} (\rho P_0+\varepsilon)^2 \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon - c(t^*, e_1) \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon\right] \quad \text{(A.19)}$$

We consider two cases:

<u>Case I</u>. (Risk Aversion). In this case, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) > 0\ \forall c_\varepsilon^*$ and $B \lesseqgtr 0$ providing that $t_n - t_0 \lesseqgtr m$, where:

$$m = \frac{c(t^*,e_1) \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left(\frac{\rho P_0+\varepsilon}{2}\right) \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon) d\varepsilon}{\frac{1}{4} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} (\rho P_0+\varepsilon)^2 \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon) d\varepsilon} > 0 \quad \text{(A.20)}$$

In this context, $|B|$ increases when $\gamma_{c_\varepsilon^*}$ goes up for all values of c_ε^*. In the screening scenario, $\frac{\partial F}{\partial t^*} > 0$ and when $t_n - t_0 < m$, then $B < 0$ and a parallel rise in $\gamma_{c_\varepsilon^*}$ will cause a drop in $\frac{\partial F}{\partial t^*}$ for all values of t^*. Thus, the value of t^* at which $\frac{\partial F}{\partial t^*} = 0$ will increase. In the signaling scenario, $\frac{\partial F}{\partial t^*} < 0$ and when $t_n - t_0 < m$, then $B < 0$ and a parallel rise in $\gamma_{c_\varepsilon^*}$ will cause a drop in $\frac{\partial F}{\partial t^*}$. Thus, the value of t^* at which $\frac{\partial F}{\partial t^*} = 0$ will go down. Clearly, when $t_n - t_0 > m$, all those effects of a parallel change in $\gamma_{c_\varepsilon^*}$ on t^* are reversed because $B > 0$.

Case II. (Risk Loving). In this case, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*}U'(c_\varepsilon^*) < 0 \; \forall c_\varepsilon^*$ and $B \lesseqgtr 0$ providing that $t_n - t_0 \gtreqless m$.

Once again, $|B|$ increases when $\gamma_{c_\varepsilon^*}$ goes up for all values of c_ε^*. When $t_n - t_0 < m$, then $B > 0$ and an increase in $\gamma_{c_\varepsilon^*}$ for all values of c_ε^* will cause a rise in B. As a result, $\frac{\partial F}{\partial t^*}$ will increase in the screening scenario and the value of t^* at which $\frac{\partial F}{\partial t^*} = 0$ will go down. In the signaling scenario, $\frac{\partial F}{\partial t^*} < 0$ and an increase in $\gamma_{c_\varepsilon^*}$ for all values of c_ε^* will cause a rise in B and $\left|\frac{\partial F}{\partial t^*}\right|$ will go down, which implies that the value of t^* at which $\frac{\partial F}{\partial t^*} = 0$ will go up. On the contrary, the sign of the effects of a change in the measure of the worker´s risk loving on t^* will be reversed in each scenario when $t_n - t_0 > m$ because $B < 0$.

This completes the proof of proposition 3. QED.

Proof of Lemma 3. As shown by assumptions 4 and 4´, the screening scenario arises when $\frac{\partial F}{\partial t^*} > 0$ and the signaling scenario appears when $\frac{\partial F}{\partial t^*} < 0$. As shown in the proof of proposition 3, $\frac{\partial F}{\partial t^*} = A + B + C$, where $A > 0, C < 0$ and $|B|$ goes up when γ_c goes up for all levels of c.

When the worker is risk-averse, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*}U'(c_\varepsilon^*) > 0$ and the sign of B depends on $t_n - t_0$. In particular, there exists $m_1 \in \mathbb{R}^+$ such that $B \lesseqgtr 0$ if $t_n - t_0 \lesseqgtr m_1$. We distinguish two cases:

Case I. When $t_n - t_0 < m_1$, then $B < 0$. In this case, there exist $\overline{\gamma}_2, \Delta_2 \in \mathbb{R}^+$ such that $A < -(B + C)$ when $\gamma_c > \overline{\gamma}_2 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_o, t_n]$. Therefore, $\frac{\partial F}{\partial t^*} = A + B + C < 0$ when $t_n - t_0 < m_1, \gamma_c > \overline{\gamma}_2 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_o, t_n]$.

Case II. When $t_n - t_0 > m_1$, then $B > 0$. In this case, there exist $\overline{\gamma}_1, \Delta_1 \in \mathbb{R}^+$ such that $A + B > -C$ providing that $\gamma_c > \overline{\gamma}_1 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1 \; \forall t \in [t_o, t_n]$. Hence, $\frac{\partial F}{\partial t^*} = A + B + C > 0$ when $t_n - t_0 > m_1, \gamma_c > \overline{\gamma}_1 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1 \; \forall t \in [t_o, t_n]$.

This completes the proof of lemma 3. QED.

Proof of Lemma 4. Once again, we use the expression of the derivative shown in the proof of proposition 3: $\frac{\partial F}{\partial t^*} = A + B + C$.

When the worker is risk-lover, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) < 0$ and there exists $m_2 \in \mathbb{R}^+$ such that $B \lesseqgtr 0$ if $t_n - t_0 \gtreqless m_2$. We distinguish two cases:

Case I. When $t_n - t_0 < m_2$, then $B > 0$. In this case, there exist $\overline{\gamma}_3, \Delta_3 \in \mathbb{R}^+$ such that $A + B > -C$ when $\gamma_c > \overline{\gamma}_3 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3 \; \forall t \in [t_o, t_n]$. Therefore, $\frac{\partial F}{\partial t^*} = A + B + C > 0$ when $t_n - t_0 < m_2, \gamma_c > \overline{\gamma}_3 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3 \; \forall t \in [t_o, t_n]$.

Case II. When $t_n - t_0 > m_2$, then $B < 0$. In this case, there exist $\overline{\gamma}_4, \Delta_4 \in \mathbb{R}^+$ such that $A < -(B + C)$ providing that $\gamma_c > \overline{\gamma}_4 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4 \; \forall t \in [t_o, t_n]$. Hence, $\frac{\partial F}{\partial t^*} = A + B + C < 0$ when $t_n - t_0 > m_2$, $\gamma_c > \overline{\gamma}_4 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4 \; \forall t \in [t_o, t_n]$.

This completes the proof of lemma 4. QED.

Proof of Proposition 4. To start with, we rewrite the indifference condition included in equation (5) as:

$$F[t^*, \delta, P_0, E(P_1|P_0), e_1] = U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] - \delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] = 0 \quad \text{(A.21)}$$

If we derive this function with respect to the indifferent type, we obtain:

$$\frac{\partial F}{\partial t^*} = \kappa P_0 \frac{\partial E(t|t < t^*)}{\partial t^*} - \delta\kappa \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big) + \delta\kappa \frac{\partial c(t^*, e_1)}{\partial t^*} \qquad \text{(A.22)}$$

Recall that $\kappa = \frac{\partial U(x)}{\partial x}$, which is constant because the worker is risk-neutral. Under assumption 7 (assumption 10), $\frac{\partial F}{\partial t^*} > 0$ ($\frac{\partial F}{\partial t^*} < 0$).

Similarly, we obtain the following derivatives:

$$\frac{\partial F}{\partial \delta} = U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] - U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \quad \text{(A.23)}$$

$$\frac{\partial F}{\partial P_0} = \kappa[E(t|t < t^*) - \delta\phi(t^*)\rho] \qquad \text{(A.24)}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*, e_1) \qquad \text{(A.25)}$$

Now, we determine the signs of these derivatives. Recall that the indifference condition included in equation (5) can be expressed as:

$$U(E(t|t < t^*)P_0) = \delta\big\{U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] -$$
$$U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big]\big\} \qquad \text{(A.26)}$$

The left-hand side of this equation is positive because $U(E(t|t < t^*)P_0) > U(t_0 P_0) > 0$. Therefore, the right-hand side of the equation is also positive, which implies that $\frac{\partial F}{\partial \delta} < 0$. It is also clear that $\frac{\partial F}{\partial e_1} > 0$. By using the implicit function theorem, we conclude that $\frac{\partial t^*}{\partial \delta} > 0$, and $\frac{\partial t^*}{\partial e_1} < 0$ in the screening scenario, whereas $\frac{\partial t^*}{\partial \delta} < 0$ and $\frac{\partial t^*}{\partial e_1} > 0$ in the signaling scenario. Finally, $\frac{\partial F}{\partial P_0}$ in equation (A.24) is positive when $\rho = 0$ and strictly decreases with ρ. Hence, there exists $K \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$, which implies that $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ when $\rho \lesseqgtr K$ in the screening scenario and $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$ in the signaling scenario. QED.

Proof of Lemma 6. We need to analyze the sign of $\frac{\partial F}{\partial t^*}$, which is shown in equation (A.22).

As shown by lemma 5, when f is an everywhere increasing function, then $\frac{\partial \phi(t^*)}{\partial t^*} < 0 \ \forall t \in [t_0, t_n]$. As a result, equation (A.22) shows that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| = 0$. Since $\frac{\partial F}{\partial t^*}$ decreases with $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right|$, there exists $c_1 \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| < c_1$.

Hence, we have proven that $\frac{\partial F}{\partial t^*} > 0$ as long as f is an everywhere increasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1 \ \forall t \in [t_0, t_n]$, which is the first part of lemma 6.

Similarly, lemma 5 shows that $\frac{\partial \phi(t^*)}{\partial t^*} > 0 \ \forall t \in [t_0, t_n]$ when f is an everywhere decreasing function. From equation (A.22), we can guarantee that $\frac{\partial F}{\partial t^*} < 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \max_{t^* \in [t_0, t_n]} \left\{\frac{P_0}{\delta} \frac{\partial E(t|t < t^*)}{\partial t^*}\right\} = c_2$.

Therefore, we have proven that $\frac{\partial F}{\partial t^*} < 0$ as long as f is an everywhere decreasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2 \ \forall t \in [t_0, t_n]$, which is the second part of lemma 6 and this completes the proof. QED.

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