Analytical Eigenstate Equivalent Circuit for Narrow-Slot Bi-Periodic Scatterers

Alberto Hernández-Escobar(1)(2), Francisco Mesa(3), Jaime Esteban(4), Elena Abdo-Sánchez(1),
Teresa M. Martín-Guerrero(1), and Carlos Camacho-Peñalosa(1)

(1) Instituto Universitario de Investigación en Telecomunicación, Escuela Técnica Superior de Ingeniería de Telecomunicación
Andalucía Tech, Universidad de Málaga, 29010 Málaga, Spain ({ahe, elenaabdo, teresa, ccp}@ic.uma.es)
(2) Department of Electrical and Electronic Engineering, Tokyo Institute of Technology, 152-8552 Tokyo, Japan
(3) Department of Applied Physics 1, E.T.S. Ing. Informática, Universidad de Sevilla, 41012 Seville, Spain (mesa@us.es)
(4) Information Processing and Telecommunications Center, Escuela Técnica Superior de Ingenieros de Telecomunicación,
Universidad Politécnica de Madrid, 28040 Madrid, Spain (jesteban@etc.upm.es)

***Abstract*—The analytical values of the parameters of an eigenstate-based equivalent circuit are obtained for the analysis and modeling of a simple bi-periodic scatterer. The scatterer geometry consists of a rotated, very narrow slot. Analytical values of this kind of equivalent circuit are obtained for the first time by comparison with a fully-analytical multi-mode equivalent circuit. Analytical results are contrasted with simulated ones, and a very good agreement is found.**

I. Introduction

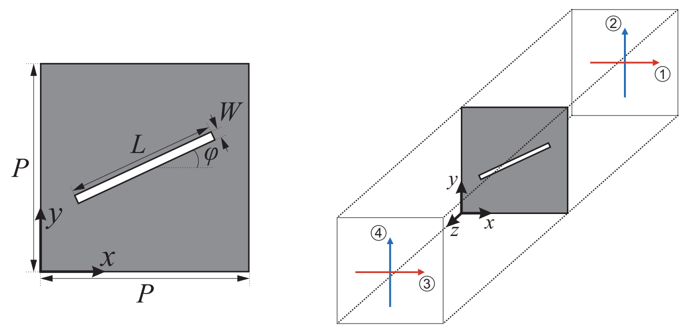
The use and analysis of bi-periodic scatterers, such as frequency selective surfaces, reflectarrays, or rasorbers, is an interesting problem for the antenna community. The analysis and synthesis of such structures are usually performed by FEM or FDTD full-wave electromagnetic simulation. However, the use of an equivalent circuit has obvious advantages, but usually needs a more in-depth study of the structure.

Some of the authors proposed in [1] an equivalent-circuit topology for asymmetric two-ports that allowed the decomposition in eigenstates. In [2], this circuit was applied to bi-periodic scatterers to obtain relatively generic equivalent circuits from one or a few electromagnetic simulations. In addition, the circuit can provide physical insight into complex structures and separate their quasi-even and quasi-odd modes. On the other hand, the circuit proposed in [3], [4] provides information on the behavior of the unit cell using analytical methods, avoiding simulations. Although this makes its use much more efficient, it can only be applied to specific geometries, since they require simplifications and hypotheses on the fields in the structure.

The purpose of this work is to find a common base in these two approaches, with the aim of obtaining analytical expressions for the circuit elements of [2] for simple geometries. As a first step, a unit cell consisting of only one narrow slot is assumed to find the relationship between both circuits. The performance of the analytical circuit is validated with simulation results. Additionally, this is a groundwork to obtain a more general equivalent circuit based on only analytical assumptions.

II. Analyzed Case: Narrow Slot

The geometry of the unit cell chosen in this work to compare the circuit approaches in [2] and [4] consists of



(a) (b)

Fig. 1. Geometry of the unit cell of the studied bi-periodic surface. (a) Unit-cell dimensions: $L = 7\,\mathrm{mm}$, $W = 0.1\,\mathrm{mm}$, $P = 10\,\mathrm{mm}$, $\varphi = 25\,°$. (b) Sketch of the polarizations used.

a narrow slot rotated 25º in a metallic plate surrounded by air, as shown in Fig. 1(a). This unit cell forms a bi-periodic surface in the xy-plane. In order to simplify the analysis even more, other assumptions are made, such as considering only normal incidence, zero-thickness metallization, and operating frequencies below the diffraction regime. Normal incidence under any angle can be considered by decomposing the two orthogonal polarizations incident to the surface (vertical and horizontal polarizations) and treating each of these polarizations as separate ports in the structure, as shown in Fig. 1(b). All these simplifications do not significantly compromise the accuracy of the analysis and allow the behavior of the unit cell to be reproduced by using a four-port circuit.

For this simplified case, the topology of the equivalent circuit obtained by analytical means in [4] is shown in Fig. 2(a). Its parameters can be obtained from the assumed frequency-independent normalized transverse electric field at the slot, $\hat{\mathbf{E}}$ as follows:

$$N_{nm}^{\mathrm{TM}} = \mathrm{FS}\{\hat{\mathbf{E}}\}_{nm} \cdot (\widehat{\mathbf{n}, \mathbf{m}}) \quad (1)$$

$$N_{nm}^{\mathrm{TE}} = \mathrm{FS}\{\hat{\mathbf{E}}\}_{nm} \cdot (\widehat{\mathbf{m}, -\mathbf{n}}) = \mathrm{FS}\{\hat{\mathbf{E}}\}_{nm} \cdot [(\widehat{\mathbf{n}, \mathbf{m}}) \times \hat{\mathbf{z}}] \quad (2)$$

$$Y_{\mathrm{eq}} = 2 \sum_{nm}{}' \left(|N_{nm}^{\mathrm{TM}}|^2 Y_{nm}^{\mathrm{TM}} + |N_{nm}^{\mathrm{TE}}|^2 Y_{nm}^{\mathrm{TE}} \right), \quad (3)$$

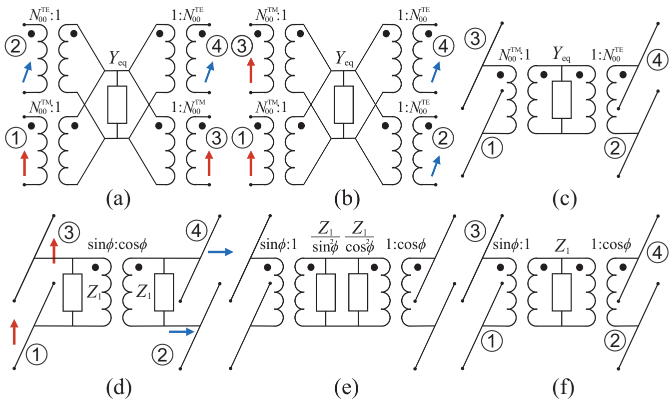


Fig. 2. Equivalent circuits from [4], (a) to (c), and [2], (d) to (f).

where $\text{FS}\{\cdot\}_{nm}$ stands for the coefficients of the bidimensional Fourier series and $\widehat{(\mathbf{n},\mathbf{m})}$ for the unitary vector of $n\hat{\mathbf{x}}+m\hat{\mathbf{y}}$. The prime symbol in the series stands for summation $\forall n,m$ except $n,m=0,0$, and Y_{nm}^{TM} and Y_{nm}^{TE} are the free-space admittances associated to the harmonics of order n,m for the TM and TE polarizations, respectively.

In the case of the equivalent circuit in [2], it is more convenient to use its impedance form [1]. The reason is that the analyzed structure does not have admittance matrix due to being an aperture in a zero-thickness metallic plate. In that case, its values can be obtained from the Z-parameters of the simulated unit cell surrounded by walls of periodic boundary conditions as

$$\phi=\frac{1}{2}\arctan\left(\frac{2z_{12}}{z_{22}-z_{11}}\right) \tag{4}$$

$$Z_1=\frac{z_{11}+z_{22}+\sqrt{(z_{11}-z_{22})^2+4z_{12}^2}}{2} \tag{5}$$

$$Z_2=\frac{z_{11}+z_{22}-\sqrt{(z_{11}-z_{22})^2+4z_{12}^2}}{2}\,. \tag{6}$$

Because the considered slot is very narrow, Z_2 is negligible, and its branch can be removed from the circuit, resulting in the one shown in Fig. 2(d).

To obtain a relationship between both circuits, several approaches can be followed. One of them consists in extracting the Z-parameters of Fig. 2(a) and using (4) and (5) obtaining the parameters of the other circuit. By doing this, the following relationships are obtained:

$$\sin\phi=N_{00}^{\text{TM}} \tag{7}$$

$$\cos\phi=N_{00}^{\text{TE}} \tag{8}$$

$$Z_1=\frac{1}{Y_{\text{eq}}}\,. \tag{9}$$

The simplicity of these relationships implies that both equivalent circuits have the same parameters, although their topology is different. Actually, using the properties of the ideal transformers, it is possible to prove these relationships in an additional way. First, the circuit in Fig. 2(a) can have its ports rearranged as in Fig. 2(b). Then, the identical transformers can be combined, as shown in Fig. 2(c). On the other side, the transformer of the circuit in Fig. 2(d) can be split into two in cascade, and the side impedances can be placed between them, as shown in Fig. 2(e). Then, the parallel of these two impedances is Z_1 due to the Pythagorean identity, leading to Fig. 2(f), which is the same as Fig. 2(c), making the relationships in (7), (8), and (9) straightforward.

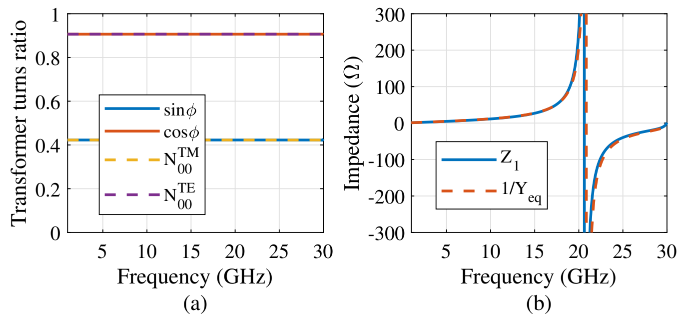


Fig. 3. Parameters of both equivalent circuits extracted from simulated results and analytical computations. (a) Transformers turns ratio. (b) Impedance.

The previous analytical results are verified by means of HFSS simulation. It is shown in Fig. 3 that the analytical parameters and the ones extracted from the simulation are almost identical and, in the same way, the S-parameters of the analytical equivalent circuit are in good agreement with the ones obtained by the simulation.

III. Conclusion

For a very simple case, analytical values of the parameters of an eigenstate equivalent circuit have been found by comparison with a multi-modal equivalent circuit. The conceptual explanation of why there is such a simple relationship between them and its application to other simple structures will be discussed in the presentation of this work.

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