New tensegrity structures based on octagonal cells

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ABSTRACT

Tensegrity families are excellent sources of new tensegrity structures. A tensegrity family is a group of tensegrity structures that share a common connectivity pattern. The Octahedron, X-Octahedron and Z-Octahedron families are examples of them. The members of these families are constructed by assembling one-bar elementary cells. Rhombic, X-rhombic and Z-shaped cells are used to construct the members of the Octahedron, X-Octahedron and Z-Octahedron families respectively. In a recent work, a new type of elementary cell was introduced: the octagonal cell. In this work, new tensegrity structures are obtained by assembling a new octagonal cell. The values of the force:length ratio of the members of the tensegrities that lead to an equilibrium configuration have been computed analytically. The stability of the resultant tensegrity structures has been analyzed.

Keywords: tensegrity, octahedron family, form-finding, force density method, octagonal cells.

1. INTRODUCTION

Tensegrity structures are free-standing pin-jointed structures composed of pre-stressed cables (tension members) and struts (compression members) that are self-equilibrated [1]. Tensegrity structures have developed greatly in recent years due to their lightweight, controllability, deployability and ingenious forms [1].

The design of tensegrity structure is a complex process because they do not exhibit very intuitive principles. Tensegrity structures have been proved to be some excellent sources of tensegrity structures. A tensegrity family is a group of tensegrity structures that share a common connectivity pattern. The Octahedron, X-Octahedron and Z-Octahedron families are examples of them. The members of these families are constructed by assembling one-bar elementary cells. Rhombic, X-rhombic and Z-shaped cells are used to construct the members of the Octahedron [2], X-Octahedron [3] and Z-Octahedron [4] respectively. In [5] a new type of elementary cell was introduced: the octagonal cell. In this work, new tensegrity structures are obtained by assembling a new octagonal cell. The values of the force:length ratio of the members of the tensegrities that lead to an equilibrium configuration have been computed analytically. The stability of the resultant tensegrity structures has been analyzed.

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2. EQUILIBRIUM AND STABILITY OF TENSEGRITY STRUCTURES

One of the key steps in the design of tensegrity structures is to find a self-equilibrated configuration (known as the form-finding process). The Force Density Method (FDM) proposed by Schek [6] is the basis of most of form-finding methods of tensegrity structures. FDM is based on the concept of force density or force:length ratio q, which is defined as the ratio between the axial force and the length of each member of the tensegrity (q is positive for cables and negative for struts).

The equilibrium equations in the FDM for a tensegrity structure of n nodes and m members are given by Eq. (1):

$$\begin{aligned} \mathbf{D} \cdot \mathbf{x} &= \mathbf{0} \\ \mathbf{D} \cdot \mathbf{y} &= \mathbf{0} \\ \mathbf{D} \cdot \mathbf{z} &= \mathbf{0} \end{aligned} \quad (1)$$

where $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ ($\in \Re^n$) are the nodal coordinate vectors, and $\mathbf{D} \in \mathbb{R}^{n \times n}$ is the force density matrix. Matrix **D** can be constructed according to Eq. (2):

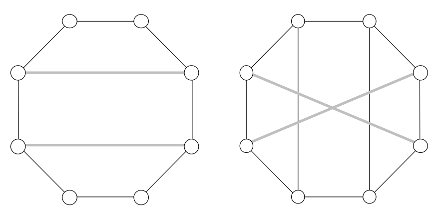
$$D_{ij} = \begin{cases} \sum_{k \in \Gamma} q_k & \text{for } i = j \\ -q_k & \text{if nodes } i \text{ and } j \text{ connected by member } k \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

In Eq. (2) is the set of members connected to the node *i*.

A tensegrity is fully developed in a *d*-dimensional space if the rank deficiency of matrix **D** is at least *d* + 1 (non-degeneracy condition) [1]. Regarding stability, a tensegrity is said stable if its tangent stiffness matrix **K** is positive semi-definite [1]. Super-stability (a most robust criterion) implies that the tensegrity is stable regardless of the material properties and prestress levels, and it must fulfill some conditions [1]. In this work, two cross-sectional areas of the members are considered (one for cables and one for struts) such that the maximum prestress in both cases equals 1% of the product *EA* (*E* = 200.000 MPa), as in [1].

3. TENSEGRITY STRUCTURES BASED ON OCTAGONAL CELLS

The members of the tensegrity families are constructed by assembling elementary cells following a certain topology. In [5], the Z-Octahedron family was defined based on a new type of elementary cell: the octagonal cell (see Fig. 1.a).



(a) (b)

Figure 1. *Octagonal cell proposed in [5] (a) and new octagonal cell (b). Gray and black lines correspond to struts and cables respectively.*

In this work, a new octagonal cell is proposed with crossed struts and two additional cables (see Fig. 1.b). Two force:length ratios are going to be considered: q_b for struts/bars and q_c for cables.

The first tensegrity to be studied is the new expanded octahedron composed by the proposed octagonal cell, which has 24 cables, 6 struts and 12 nodes. The plane connection graph of this new tensegrity is shown in Fig. 2. This plane connection graph has been obtained by replacing the octagonal cell shown in Fig. 1.a by the new one shown in Fig. 1.b in the plane connection graph proposed in [6]. The connectivity between the nodes is one of the inputs of the form-finding method (the other is the force:length ratio assignation for the members composing the tensegrity structure).

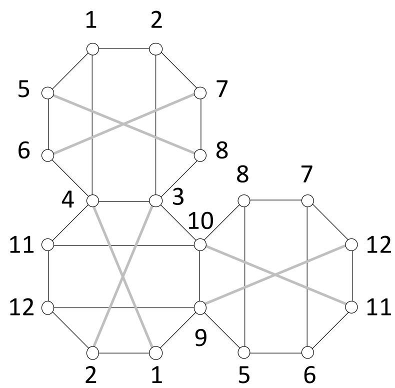


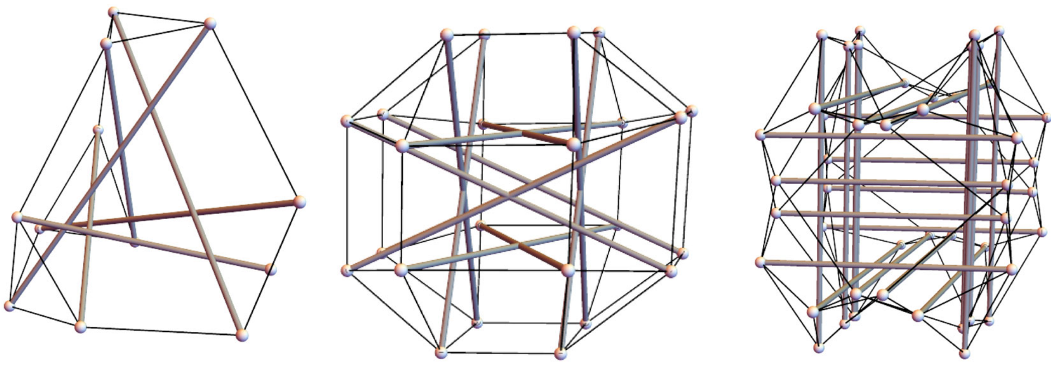
Figure 2. *Plane connection graph of the expanded octahedron composed by the new octagonal cells shown in Fig. 1. Gray and black lines correspond to struts and cables respectively.*

For the expanded octahedron case, two solutions are obtained: $q_b = \frac{1}{8}\left(-19\,q_c - \sqrt{41}\,q_c\right)$ (unstable) and $q_b = \frac{1}{8}\left(-19\,q_c + \sqrt{41}\,q_c\right)$ (super-stable). The equilibrium shape of the super-stable solution is shown in Fig. 3.a.

The double-expanded octahedron composed by the octagonal cell shown in Fig. 1.b has 48 cables, 12 struts and 24 nodes (twice the number of cables, struts and nodes than the expanded octahedron). The connectivity of the new double-expanded octahedron has been obtained as in the previous case: by replacing the octagonal cell shown in Fig. 1.a by the new one shown in Fig. 1.b in the corresponding plane connection graph proposed in [5]. The plane connection graph has not been represented in this work due to size limitations. In this case, four solutions are obtained: $q_b = \frac{1}{8}\left(-19\,q_c - \sqrt{41}\,q_c\right)$ (unstable) and $q_b = \frac{1}{8}\left(-19\,q_c + \sqrt{41}\,q_c\right)$ (stable), $q_b = \frac{1}{8}\left(-11\,q_c - \sqrt{41}\,q_c\right)$ (unstable), and $q_b = \frac{1}{8}\left(-11\,q_c + \sqrt{41}\,q_c\right)$ (super-stable). It has to be pointed out that the solutions corresponding to the expanded octahedron case are also solutions of the form-finding problem in the double-expanded octahedron case. This is because the expanded octahedron is a folded form of the double-expanded octahedron. The equilibrium shape of the super-stable solution is shown in Fig. 3.b.

Finally, the triple-expanded octahedron composed by the octagonal cell shown in Fig. 1.b has 96 cables, 24 struts and 48 nodes (twice the number of cables, struts and nodes than the double-expanded octahedron). The plane connection graph has not been represented in this work due to size limitations. In this case, four solutions are obtained: $q_b = \frac{1}{8}\left(-19\,q_c - \sqrt{41}\,q_c\right)$ (unstable) and $q_b =$

$\frac{1}{8}\left(-19\ q_c + \sqrt{41}\ q_c\right)$ (unstable), $q_b = \frac{1}{8}\left(-11\ q_c - \sqrt{41}\ q_c\right)$ (unstable), $q_b = \frac{1}{8}\left(-11\ q_c + \sqrt{41}\ q_c\right)$ (unstable), $q_b = \frac{1}{161}\left(-270\ q_c - \sqrt{31201}\ q_c\right)$ (unstable), and $q_b = \frac{1}{161}\left(-270\ q_c + \sqrt{31201}\ q_c\right)$ (stable). The equilibrium shape of the stable solution is shown in Fig. 3.c.



(a) (b) (c)

Figure 3. *Expanded (a), double-expanded (b) and triple-expanded (c) octahedrons composed by the new octagonal cell shown in Fig. 1.b. Gray and black lines correspond to struts and cables respectively.*

4. CONCLUSIONS

New tensegrity structures can be defined based on the cell substitution design method. In this work, a new octagonal cell formed by 10 cables and 2 struts has been defined. Three new tensegrities have been defined: the expanded octahedron, the double-expanded octahedron and the triple-expanded octahedron composed by the new octagonal cell. The first two are super-stable, and the third one is stable. It has been proved that tensegrity families are a great source of stable and super-stable tensegrity structures.

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