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American Annals of the Deaf, Volume 163, Number 3, Summer 2018, pp. 374-393 (Article)

Published by Gallaudet University Press
DOI: *https://doi.org/10.1353/aad.2018.0024*

Rodríguez-Santos, J. M., García-Orza, J., Calleja, M., Damas, J., & Iza, M. (2018). Nonsymbolic comparison in deaf students: No evidence for a deficit in numerosity processing. *American Annals of the Deaf, 163*(3), 374–393.

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It is commonly found that deaf and hard of hearing (DHH) students experience delayed mathematical achievement. The present study used two nonsymbolic comparison tasks to explore the basic numerical skills of DHH students. Nine prelocutive DHH students with cochlear implants and nine hearing students, matched on nonverbal IQ, visual short-term memory, and verbal comprehension, were recruited. The participants performed two different collection comparison tasks with different ratios and under different perceptual conditions. Analyses by task showed similar response times, accuracy, and ratio effects for both groups on the Low Perceptual Condition task, a finding suggesting that the two groups accessed similar representations of quantity. Differences in performance on the simpler High Perceptual Condition task, on which the DHH group showed slower response times, probably were strategic in origin. The results suggest that DHH students have no deficits in basic numerical skills.

KEYWORDS: deafness, numerosity, quantity processing, basic numerical skills, mathematical achievement

THE PRESENT study investigated whether deaf and hard of hearing (DHH) students experience difficulties with nonsymbolic comparison tasks (e.g., deciding which array has more elements). These tasks that assess basic quantitative processes are considered a measure of the number sense, or approximate number system (ANS). The ANS is considered a biologically determined system that is used to understand, approximate, and manipulate nonsymbolic quantities (Dehaene, 2001). Moreover, according to different models of mathematical development, this system forms the core of numerical skills acquisition (Butterworth, 2010; Dehaene, 1997, Feigenson, Dehaene, & Spelke, 2004; Gelman & Gallistel, 1978). This view is supported by the capacity of early measures of the ANS to predict individual differences in mathematical achievement (Libertus, Feigenson, & Halberda, 2011). Whereas the ANS is considered innate, the understanding and manipulation of (nonsymbolic and symbolic) numerical representations is affected by several factors, including socioeconomic status (Butterworth, 2010). Thus, a starting point in understanding the difficulties DHH students usually experience with mathematics would be to investigate whether the reduced exposure to information usually associated with deafness affects the integrity of the ANS.

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Although the diverse and heterogeneous nature of the DHH population requires that any finding be evaluated with caution, research has consistently shown that DHH students perform poorly on mathematical tasks (e.g., Allen, 1995; Ansell & Pagliaro, 2006; Kritzer, 2007; Leybaert & Van Cutsem, 2002; Marschark & Everhart, 1999; Mousley & Kurz, 2015; Pagliaro & Kritzer, 2013; Rodríguez-Santos, Calleja, García-Orza, Iza, & Damas, 2014). When standardized tests are conducted in school settings, studies suggest that there is a delay in mathematical performance of 2.0–3.5 years among DHH students relative to hearing students (for reviews, see Kritzer, 2007; Nunes, 2004; Pagliaro, 2010; Traxler, 2000). These studies also suggest that the delay shown by DHH students is constant, and that this gap, which appears early in life, does not decrease in subsequent years. As noted by several authors, DHH students exhibit mathematical difficulties across a wide age range, showing deficits in mental calculation, arithmetical problem solving, logical reasoning, and understanding of fractional concepts (Allen, 1995; Ansell & Pagliaro, 2006; Marschark & Everhart, 1999; Mousley & Kurz, 2015). The difficulties experienced by DHH students with these mathematical tasks are relevant because such tasks form part of their daily activities and will affect their future socioeconomic and professional integration.

DHH students' reported difficulties in mathematical achievement are generally explained by (a) their limited access to wide-ranging numerical experience and (b) the low-level language they often use (Gregory, 1998; Kritzer, 2009; Nunes, 2004; Pagliaro, 2010; Pixner, Leyrer, & Moeller, 2014),[1] or the kind of language teachers use with them. Madalena, Silva, Santos, and Marins (2015) showed that DHH students who had early exposure to a sign language (Brazilian Sign Language) had better results than those with a late exposure to the same language. Along the same lines, as Pixner et al. (2014) have stated:

> It seems that . . . children with [cochlear implants] are able to acquire arithmetic skills in a qualitatively similar fashion [to that of] their normal[ly] hearing peers. Nonetheless, when demands [are put] on place-value understanding, which has only recently been proposed to be language mediated, hearing impaired children experience specific difficulties. (p. 1)

Regarding the first factor, limited access, it has been suggested that hearing children typically engage in more daily activities that involve the use of numbers than their deaf peers (Kritzer, 2009). Research on typically hearing families has suggested that parents, even unconsciously, increase their children's potential for learning by making spontaneous use of techniques such as questioning, asking for clarification, or providing additional information (Anderson, 1997). In fact, a study by Levine, Suriyakham, Rowe, Huttenlocher, and Gunderson (2010) done with hearing participants showed an association between the use of numbers in parents' conversation with their children and the children's development of numerical knowledge (see also Pagliaro & Kritzer, 2010). DHH children have problems with accessing oral language (Dahl et al., 2003). This is even true, though to a lesser extent, for those DHH children who receive a cochlear implant (CI) at a young age (Geers, Moog, Biedenstein, Brenner, & Hayes, 2009) and, in general, for DHH children of hearing parents without knowledge of sign language (Mitchell & Karchmer, 2004). Thus, it is likely that these children's communicative difficulties reduce the probability of occurrence of informal and natural interactions involving numerical knowledge.

These communicative difficulties may also affect DHH children's exposure to numerical knowledge in the school setting. Kelly, Lang, and Pagliaro (2003) explored the strategies followed by mathematics teachers of deaf students, and found that deaf students in grades 6–12 experienced reduced exposure to numerical concepts. Kelly et al. also found that the problem-solving strategies offered to hearing students were more diverse than those offered to DHH students, pointing to the fact that strategies employed by DHH students' instructors focus more on practice exercises (calculation) than on true problem-solving situations. Consequently, these teachers fail to make connections to the informal language used by their students' parents in daily life activities.

Regarding low-level language, it has been demonstrated that language (oral or signed) plays a crucial role in the acquisition of number concepts in general (e.g., Carey, 2004; Göbel, Moeller, Pixner, Kaufmann, & Nuerk, 2014; LeFevre et al., 2010; Madalena et al., 2015). There is evidence of the role of oral language in the learning of specific tasks such as retrieving arithmetical facts—for instance, in the case of single-digit multiplication (Dehaene, 1992; Lee & Kang, 2002). Therefore, language difficulties appear to be involved in the acquisition of mathematical concepts and skills. Nevertheless, the growing use of CIs has clearly improved the speech and oral-language skills of DHH students. It is beyond doubt that CIs have led to clear progress in the use of oral language (Archbold & Mayer, 2013; for critical reviews, see Marschark, Sarchet, Rhoten, & Zupan, 2010; P. E. Spencer, Marschark, & L. J. Spencer, 2011). However, studies have suggested that CIs are far from sufficient as a means of providing a natural, fully accessible auditory experience, and that difficulties typically associated with hearing loss, including access to mathematics, may therefore remain (Geers, et al., 2009; Marschark et al., 2010).

Learning practices in school are another factor that may add to the difficulties experienced by DHH students. Pagliaro (2010) analyzed the formal instruction received by DHH students in different school settings and found that the emphasis on memorization, repetition, and practice worksheets rarely allowed DHH students to learn in an effective way. Pagliaro suggested that this type of instruction clearly contributes to the poor mathematics performance of DHH students. Nunes et al. (2009) found that the performance of DHH children was worse than that of hearing children on a multiplicative reasoning task in the first 2 years of schooling. Nevertheless, Nunes et al. also noted that DHH children's performance significantly improved after a short training period based on a step-by-step analysis of how to use correspondences to solve multiplicative reasoning tasks. Thus, the researchers concluded, instruction must be adapted to the needs of DHH children to provide a firmer basis for learning mathematics.

The Search for a Deficit in Basic Numerical Skills in Students Who Are DHH

As noted above, DHH students only partially master the knowledge and skills needed to do mathematics correctly, and different factors, not limited to language, appear to play a role in this problem. A central question in studies of deficits in the mathematical skills of DHH students is whether these students show deficits in basic numerical skills, such as estimating sets of elements like dots, sticks, or circles, or comparing Arabic numbers (e.g., Bull et al., 2011; Kritzer, 2009; Pagliaro & Kritzer, 2013; Rodríguez-Santos et al., 2014), or whether, on the contrary, these abilities are spared and DHH students only struggle with more complex tasks, including those that involve language, such as mental calculation or arithmetical problem solving (e.g., Arfé et al., 2011; Nunes et al., 2009).

This is not a trivial question because a dominant view in numerical cognition studies is that the development of mathematical skills depends on the integrity of the ANS, an approximate analogue system that computes large numerosities (Dehaene, 1997, 2001). According to some models, the ANS would be different from an individuation system that was capable of subitizing—that is, precisely perceiving small quantities at a glance (the subitizing range being 1–4; Feigenson et al., 2004).

Several studies have suggested that the development of symbolic numerical knowledge relies on the representation of approximate numerosities, and that this ability increases with age and throughout the school years (Halberda, Mazzocco, & Feigenson, 2008; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). Other studies have also suggested that individual differences in ANS acuity are predictive of arithmetic ability (Bugden & Ansari, 2011; Gebuis & Reynvoet, 2015; He et al., 2016). Park and Brannon (2013) found that training to improve ANS acuity leads to better performance in symbolic mathematics. In support of these findings, it has been proposed that dyscalculia, a mathematics-specific learning disability, has its origin in an impairment of the ANS (Butterworth, 2010; Piazza et al., 2010; but see also Kauffman et al., 2013, for a different view). The *defective number module* hypothesis suggests that poor ANS acuity would underlie the difficulties in the development of mathematical skills in children with dyscalculia. In contrast, the *access deficit* hypothesis suggests that difficulties in learning mathematics are caused by a deficit in accessing numerical representations from symbolic notations—for example, Arabic “4” or verbal “four” (Rousselle & Noël, 2007; see also Defever, De Smedt, & Reynvoet, 2013, and De Smedt & Gilmore, 2011). The defective number module hypothesis suggests that as a result of their poor numerical representations, children with dyscalculia can be expected to have difficulty performing tasks such as comparing collections of elements like dots, squares, circles, or sticks, which are a classic measure of the ANS, as well as tasks in which Arabic numbers are compared (symbolic tasks). In contrast, the access deficit hypothesis suggests that the performance of children with dyscalculia should remain unaffected in nonsymbolic comparison tasks, whereas their performance should worsen only when they compare Arabic digits.

By basing the present study on developmental dyscalculia (DD) theories, we do not mean to imply that DHH children necessarily have dyscalculia, a developmental disability; rather, we propose that reduced exposure of DHH children to numerical concepts can lead to similar problems (i.e., to deficits in ANS acuity or in accessing numerical representations from symbolic notations). A review of the literature on DD shows that it is a matter of research whether DD represents the extreme end of a continuum (or several continua) of mathematical ability or whether the arithmetic difficulties associated with DD are qualitatively different from more common mathematics difficulties. There is evidence to support either of these positions (see Figure 1 in Kaufmann et al., 2013; see also Rubinstein & Henik, 2009). Therefore, a subject of investigation should be whether DHH children would experience difficulties in foundational numerical tasks as a consequence of having reduced exposure to information and more limited opportunities for incidental learning (Kritzer, 2009, Levine et al., 2010; Pagliaro & Kritzer, 2010). Most of the available literature suggests that DHH individuals have difficulties with simple tasks that require the use of symbolic representations, that is, Arabic or verbal representation of numbers (e.g., Bull, Marschark, & Blatto-Vallee, 2005; Bull et al., 2011; Kritzer, 2009; Leybaert & Van Cutsem, 2002; Rodríguez-Santos et al., 2014). However, because most of these studies have investigated numerical representations using symbolic materials, it is not possible to determine whether the difficulties experienced by DHH students are due to problems in numerical representations, a finding that would be in line with the defective number module hypothesis, or to problems accessing these representations from the symbolic notation, a finding that would be consistent with the access deficit hypothesis. A review of the literature shows that studies on the nonsymbolic abilities of DHH children remain scarce.

Bull, Blatto-Vallee, and Fabich (2006) investigated the ability of DHH students and hearing adults to rapidly enumerate small sets of dots (ranging in number from one to six). The dots were presented in three conditions: arranged in two groups, arranged canonically, and arranged randomly. No differences were found between groups in their numerical representations. However, a closer examination of the data (see Figure 2 in Bull et al., 2006) showed that DHH adults displayed less accuracy when sets of five or six dots were randomly presented. (It should be noted that the means and the corresponding *p* values for this comparison were not provided because the critical interaction presentation Format x Number Range x Hearing Group was not significant.) This finding suggests that differences could exist between groups in regard to quantities above the subitizing range (1–4).

Zarfaty, Nunes, and Bryant (2004) assessed the ability of DHH children and hearing children (age range 3–4 years) to reproduce sets composed of two, three, or four blocks. Blocks were presented simultaneously or sequentially. No differences between groups were found in the sequential condition, but the DHH group performed better than the hearing group in the simultaneous condition. This finding has been explained by the advantage DHH students have in some spatial tasks (see Emmorey, 1998).

Similar results were found by Arfé et al. (2011) in a study comparing quantities between one and nine. The researchers assessed the performance of 10 DHH children (age range 4–6 years) with CIs to that of hearing children of a similar age on digit and dot comparison tasks. No difference was found between DHH children and hearing children in the Arabic comparison task, but the DHH group outperformed the hearing group in the dot comparison task.

In a similar study with DHH children (age range 8–10 years), Rodríguez-Santos et al. (2014) found no differences in a dot comparison task with quantities ranging from 1 to 10. However, DHH children showed a delay when comparing Arabic digits. In line with the access deficit hypothesis, Rodríguez-Santos et al. concluded that although numerical representations were well formed in the DHH students, access from symbolic notation was not well automated.

The studies by Arfé et al. (2011), Bull et al. (2006), Rodríguez-Santos et al. (2014), and Zarfaty et al. (2004) suggested that DHH participants build nonsymbolic representations that were as robust as, or even better, than those of hearing participants. However, very low quantities (no greater than 10) were employed in these nonsymbolic tasks. Furthermore, the procedure used was different from the usual one for studies on the ANS among children and adults. It is generally accepted that quantity representations within the ANS become more approximate as the quantity increases (Dehaene, 1997; see Clayton, Gilmore, & Inglis, 2015, for a review). Hence, a wider range of quantities should be used to investigate the ANS. In addition, most of these studies did not take into account the role of perceptual factors. Mix, Huttenlocher, and Levine (2002) suggested that dot comparison tasks can be solved by relying on perceptual information, such as the amount of area covered by dots of similar size or the density of the dot collections. These perceptual cues correlate with numerosity, and it is therefore difficult to determine which information the participant relies on when solving the task. Although there is a heated debate in the current literature (see, e.g., Clayton et al., 2015; Gebuis & Reynvoet, 2012) on the role played by perceptual factors and how they can be controlled, it is clear that a more precise measure of the ANS could be obtained by creating conditions in which some of these factors were controlled and participants were forced to rely on quantity. To date, the evidence in favor of intact ANS representations in DHH students does not seem very strong: Some data are ambiguous, the influence of perceptual cues has in general not been controlled, and only small numbers of dot collections have used in testing.

Although attributable to other factors, a similar conclusion about uncertain evidence can be made by analyzing the scarce findings in support of the presence of a deficit in quantity representations (i.e., the ANS). Bull et al. (2010) compared DHH children and hearing children (mean age 9.7 years, *SD* = 1.7) in a dot comparison task in which the area was controlled. They found that the DHH children's acuity was significantly worse than the hearing children's. Interestingly, Bull et al. also found a correlation between acuity scores and performance on standardized mathematics achievement tests in both samples. Some of the same researchers participated in another study, this time with undergraduate students, that showed similar results (Marschark, Blatto-Vallee, Bull, & Cornoldi, 2003, cited in Bull et al., 2005). DHH participants were slower than hearing participants on a numerosity task and a length judgement task, a finding that supports the hypothesis that a deficit exists in magnitude and quantity processing. Unfortunately, these data have not been published in detail, so it is uncertain to what extent they are reliable.

The Present Study

In line with previous work by Rousselle and Noël (2007) with children with DD, we asked, as our research question, whether DHH children show a deficit in numerosity processing (i.e., a deficiency in their ANS). We tested whether DHH participants differed from a control group of hearing participants, matched by education, cognitive level (as indicated by visual short-term memory and IQ), and Spanish written-language comprehension (as shown by performance on a sentence-picture matching task), in two collection (nonsymbolic) comparison tasks that differed in difficulty. Collection comparison tasks are the most commonly used tasks for measuring the ANS (see, e.g., De Smedt, Nöel, Gilmore, & Ansari, 2013, and Dietrich, Huber, & Nuerk, 2015, for reviews). In one of these tasks, the *High Perceptual Condition* (HPC), the stimuli were controlled by density (there was a similar distance between the collection of sticks to be compared), although there was a correlation between numerosity and a number of perceptual features, such as total area, total surface, and brightness. In the other task, *Low Perceptual Condition* (LPC), the surface covered by the items was controlled by using sticks of different sizes, brightness, and total area. Thus, only some of the visual cues were correlated with numerosity (see Rousselle, Palmers, & Noël, 2004). Since more perceptual cues correlated with numerosity in the HPC than in the LPC, better results were expected in the HPC. In other words, in the HPC, numerical judgments can be based on numerosity and perceptual cues, whereas in the LPC, judgments should be based only on numerosity, which thus makes the LPC a better measure of the ANS (see Rousselle & Noël, 2007). Two ratios were investigated, 1/2 and 2/3, and quantities between 6 and 28 were employed. Thus, we extended the range of quantities that had been investigated in previous studies with deaf children (Arfé, et al., 2011; Bull et al., 2006; Rodríguez-Santos et al., 2014; Zarfaty et al., 2004) and avoided confounders related to the simultaneous comparison of quantities within and outside the subitizing range. We used these quantities to measure the ANS (see Dietrich et al., 2015).

According to the defective number module hypothesis, if a deficit existed in the ANS in DHH children, possibly associated with their poorer experience with mathematics (Pagliaro, 2010), these children would be expected to perform worse than typically hearing children on both tasks. In addition, differences would be expected to be found between the two groups in ratio effects, particularly in the LPC, in which there is an increased demand on the ANS. In contrast, if no deficit existed—for example, if the deficit were limited to access to symbolic codes—no differences would be expected between groups in relation to tasks or ratio. In fact, a review of the literature suggests that because of their superior visual processing, deaf children may even compare collection of dots faster and more accurately than their hearing peers (Arfé et al., 2011; Zarfaty et al., 2004).

Method

Participants

The DHH participants ($n = 9$), all Spanish speakers, comprised six boys and three girls (age range 8.4–9.7 years; $M = 8.9$, $SD = 0.38$) with congenital hearing loss. Another DHH child participated in the present study; however, he was not included in the analysis because he misunderstood the instructions for the experimental tasks. At the time of testing, the children were studying at a DHH student center attended by both DHH and hearing students, with the DHH students receiving additional support from a teacher in the form of classes 2–4 hours per week. All of the children had been trained in oral language under the Cued Speech system and a bimodal system (a nonformal combination of spoken and sign language).[2] The participants had no other disabilities. None of their parents were deaf. All of the children came from families that spoke only Spanish. The parents communicated with their children using oral language with additional support from signs. Table 1 presents further data on sociodemographic characteristics of the DHH sample.

The hearing participants, nine children with typical language development, without neurological or developmental difficulties, and without socioemotional behavior problems, were selected from a population of 50 typically hearing children. They were attending a state school serving a middle-class population, and thus were similar socioeconomically to the DHH participants. Each hearing participant was matched, as closely as possible, to a deaf

Table 1. Sociodemographic Attributes of the Study Participants (N = 18)

Deaf and hard of hearing participants (*N* = 9)								Hearing participants (*N* = 9)				
Age at testing: years, months	**Gender**	**Degree of hearing loss (dB, unaided)**	**Device used at present**	**Age at start of use of CI or HA: years, months**	**TONI-2 IQ**	**Visual memory span (VPT)**	**CELF-4 sentence comprehension test (modified) score (out of 30)**	**Age at testing: years, months**	**Gender**	**TONI-2 IQ**	**Visual memory span (VPT)**	**CELF-4 sentence comprehension test (modified) score (out of 30)**
8y 8m	M	>90 dB	CI	4y 7m (LE) 2y 1m (RE)	79	5.33	30	8y 11m	M	96	5.66	29
9y 1m	F	75 dB RE 90 dB LE	HA	3y	106	4.33	29	9y 6m	F	108	5.33	26
8y 7m	M	90 dB RE 75 dB LE	CI	5y 5m (RE)	87	3.00	21	8y 1m	M	89	4.00	22
8y 9m	M	70 dB RE 80 dB LE	HA	3y	120	6.33	27	8y 9m	M	120	5.33	27
8y 11m	F	>90 dB	HA/CI	2y 11m	82	4.00	20	9y 1m	F	89	5.00	28
8y 5m	M	>90 dB	CI	6y 5m (LE) 2y (RE)	122	5.66	22	8y 1m	M	122	7.66	24
9y 9m	M	>90 dB	CI	11m (LE) 4y 11m (RE)	84	7.33	29	9y 0m	M	89	5.33	27
9y 1m	F	>90 dB	CI	2y (LE) 6y (RE)	144	5.00	29	8y 10m	F	144	4.66	27
8y 11m	M	>90 dB	HA	2y	146	7.66	30	8y 8m	M	146	6.33	25

Notes. CI: cochlear implant. HA: hearing aid. RE: right ear. LE: left ear. TONI: Test of Nonverbal Intelligence. VPT = Visual Patterns Test. CELF: Comprehensive Evaluation of Language Fundamentals.

participant by age, and then, on the basis of his or her score on assessments of nonverbal IQ, visual short-term memory, and language comprehension by means, respectively, of the Spanish version of the Test of Nonverbal Intelligence–2 (TONI-2; Brown, Sherbenou & Johnsen, 1995), the Visual Patterns Test (VPT; Della Sala, Gray, Baddeley & Wilson, 1997), and the sentence comprehension test of the Spanish edition of the Clinical Evaluation of Language Fundamentals–4 (CELF-4; Semel, Wiig, & Secord, 2006); see Table 1. As can be observed in Table 2, no between-group differences existed in these measures. Groups were also compared on a fluency task with single-digit addition problems (e.g., 4 + 9 = __). Participants were given a booklet and were required to write the solution to as many problems as they could in 2 minutes. As has also been found in other studies (e.g., Allen, 1995; Davis & Kelly, 2003), the performance of the DHH group was less accurate than that of the hearing group in arithmetic problem solving (see Table 2).

Materials

Psychometric Tests

Three tests were used to match individuals in the DHH and typically hearing groups. The TONI-2 (Brown et al., 1995) provides a measure of nonverbal intelligence for people between the ages of 5 and 85 years. The internal consistency of the Spanish version is .89. The VPT (Della Sala et al., 1997) is a measure of visual short-term memory. It has a good test-retest reliability of .75 and an internal consistency coefficient of .81. The sentence comprehension test of the CELF-4 (Semel et al., 2006) provides an evaluation of oral sentence comprehension for people between the ages of 5 and 21 years. Its internal consistency coefficient is .74, and its intercorrelation with the Receptive Language Index is .82.

Experimental Tasks

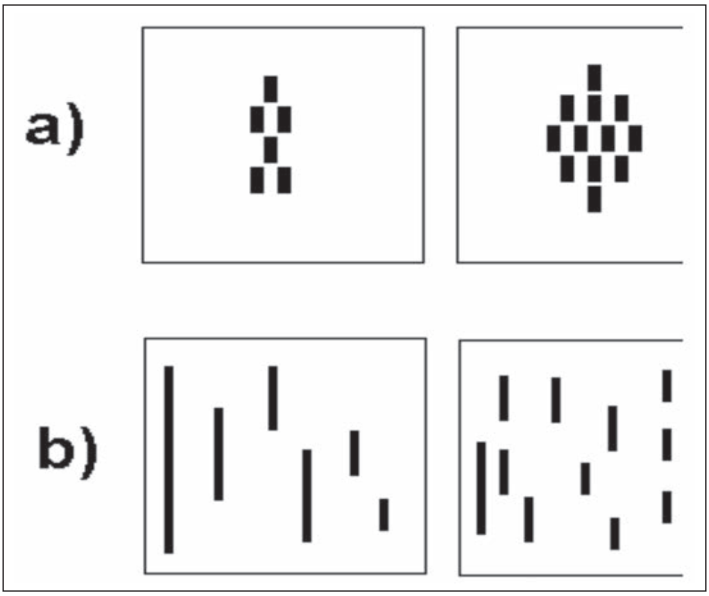
As we note in the introductory section of the present article, previous studies have suggested that perceptual variables that correlate with numerosity can influence the behavior of participants in collection comparison tasks (e.g., Mix et al., 2002; Rousselle et al., 2004; Xu & Spelke, 2000). In line with the procedure developed by Rousselle et al. (2004), the influence of perceptual variables was controlled by the use of two computerized comparison tasks in which sticks were used: the HPC task and the LPC task. These tasks were adapted from those used by Rouselle et al. (2004), who called them the Density task and the Surface task, respectively. The ratios and quan-

Table 2. Comparison Between Typically Hearing and Deaf and Hard of Hearing (DHH) Children on IQ, Visual Memory, Language Comprehension, and Arithmetic Fluency

Group	Nonverbal IQ (TONI-2)	Visual memory span (VPT)	Language comprehension (CELF-4)	Arithmetic fluency task[a]
	M (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
DHH	107.78 (26.50)	5.40 (1.53)	26.33 (4.12)	40.89 (23.50)
Hearing	111.44 (22.90)	5.48 (1.04)	26.11 (2.15)	60.18 (10.90)
	$t(16) = 0.31$ $p = .76$	$t(16) = 0.12$ $p = .91$	$t(16) = -0.14$ $p = .89$	$t(16) = 2.23$ $p = .04$

Notes. DHH group, $n = 9$. Hearing group, $n = 9$. TONI: Test of Nonverbal Intelligence. VPT = Visual Patterns Test.
CELF: Comprehensive Evaluation of Language Fundamentals.
[a] M = mean number of correctly solved problems out of 72.

Figure 1. Pairs of Stimuli: (a) High Perceptual Condition and (b) Low Perceptual Condition for a Ratio 1/2 Condition



tities we chose to employ were in line with those used by Rousselle and Nöel (2007). Their study included children whose age range was similar to that of the children in the present study, and their findings showed clear effects of ratio and perceptual condition.

In the HPC task, the distance between sticks was controlled so that it was always the same in the collections to be compared. All the sticks used in this task were black (width 2 mm, length 6 mm). The other perceptual cues (total area, surface covered by the sticks, and brightness) were constant and correlated with numerosity. In the LPC task, the perceptual variables were controlled in such a way that there was no correlation between quantity and the surface covered by the sticks, and, therefore, the total brightness (see Figure 1). We used sticks of eight different sizes to match the surface occupied by the sticks in the pair of collections to be simultaneously compared. All the sticks were the same width (2 mm) but differed in length (9–69 mm). However, there was a correlation between density and the quantity of sticks in each collection.

In both experimental tasks (HPC and LPC), we followed the procedures used by Rousselle and Noël (2007) to build six pairs of collections that varied in ratio (1/2 vs. 2/3). In the 1/2 ratio the pairs were 6/12, 8/16, and 14/28; in the 2/3 ratio the pairs were 6/9, 10/15, and 16/24. In both conditions, the display was composed of two white squares (5.5 cm per side) 8 mm apart. The collections of sticks to be compared were presented inside the white squares.

Procedure

In the first session, the psychometric tests were given to all the children in the following order: the TONI-2, the sentence comprehension test, and the VPT. The TONI-2 and VPT were administered according to the instructions provided in the test manuals. Instructions were provided in Spanish with the additional support of Spanish Sign Language. The sentence comprehension test procedure of the CELF-4 was modified in such a way that it could be used with DHH participants. In the original version of the test, four pictures are presented and a sentence is read to the children. The children are then asked to point to the picture that matches the meaning of the sentence. In the present study, sentences were presented as written statements to the DHH and hearing groups, and participants were requested to read the sentence aloud before pointing to the chosen picture.

The three tests included practice trials, so that during the practice session we confirmed that the children understood the instructions provided for each test. The whole session lasted approximately 60 minutes, with a 5-minute rest between each test.

In the second session, the HPC and LPC tasks were performed in different blocks, in which the block order was balanced across participants. Stimuli were presented and response times (RTs) recorded by computers using ERTS software (Beringer, 1999) running on MS-DOS. The same procedure was followed in both conditions. First, each trial presented a fixation point (+) for 500 ms. Next, the fixation point disappeared, and the pairs of collections of black sticks simultaneously appeared on the two white squares situated on the left and right sides of the black screen until the participant made a response. Participants were asked to press the right or left button of the keyboard according to the side of the screen on which the larger collection appeared. Participants were instructed to make this decision as rapidly and as accurately as possible. RTs were measured from target onset until the participant's response. Each pair of collections was presented four times during the experiment: twice with the larger collection positioned on the right side of the screen and twice with the larger collection on the left. Thus, there were 24 stimuli in each experimental task. Each participant performed the trials in a different order and received a total of 8 practice trials prior to the 24 experimental trials. Although the practice trials used the same manipulations as those in the experimental trials, different ratios were used (6/7, 7/8, 8/9, 5/8, 5/9, 6/9). These stimuli were also presented as fillers during the experiment to prevent the same trial from appearing two times consecutively. Thus, each participant performed 54 trials (experimental task + fillers + practice trials). Comparison pairs were presented in a fixed pseudorandom order. The experiment lasted approximately 10 minutes. Each participant received a different randomized order of trials.

RESULTS

According to the literature, there are different ways of analyzing the results of nonsymbolic comparison tasks. In line with the method used by Rousselle and Noël (2007), the present study analyzed RT and response accuracy, taking ratio and task as independent variables. Thus, analyses of variance (ANOVAs) were conducted on RT and accuracy (proportion of errors), with task (HPC, LPC) and ratio (1/2 vs. 2/3) as within-participants factors and group (DHH, hearing) as the between-participants factor. Partial eta-square (η_p^2) values were computed as a measure of effect size.

Analysis of Response Times

The RTs of correct responses alone were taken into account. Times outside the 300–3000 ms range, which represented about 1.7% of the data, were eliminated from the

Table 3. Response Times and Proportion of Errors on the HPC and LPC Tasks

	Mean response times (ms)			
	HPC		**LPC**	
Group	**Ratio 1/2** *M* (*SD*)	**Ratio 2/3** *M* (*SD*)	**Ratio 1/2** *M* (*SD*)	**Ratio 2/3** *M* (*SD*)
DHH	863 (184)	1147 (237)	992 (206)	1189 (296)
Hearing	784 (117)	941 (202)	965 (228)	1193 (286)
	Mean proportions of errors			
	HPC		**LPC**	
Group	**Ratio 1/2** *M* (*SD*)	**Ratio 2/3** *M* (*SD*)	**Ratio 1/2** *M* (*SD*)	**Ratio 2/3** *M* (*SD*)
DHH	.03 (.06)	.07 (.09)	.15 (.17)	.24 (.15)
Hearing	.01 (.01)	.09 (.06)	.10 (.22)	.27 (.18)

Notes. DHH (deaf and hard of hearing) group, *n* = 9. Hearing group, *n* = 9. HPC = High Perceptual Condition.
LPC = Low Perceptual Condition.

analysis. Table 3 shows the mean RTs for each condition in each task.

A 2 x 2 x 2 ANOVA was conducted with task and ratio as within-subject factors and group as the between-subject factor. We found main effects of task, $F(1, 16) = 11.63$, $p < .01$, $\eta_p^2 = .42$, showing that responses in the HPC task were faster than responses in the LPC task. We also found a main effect of ratio, $F(1, 16) = 30.38$, $p < .001$, $\eta_p^2 = .61$, showing that responses were faster in the 1/2 ratio. Although the DHH participants had slower RTs than the hearing participants, this difference was far from significant: $F(1, 16) = 1.95$; $p = .18$, $\eta_p^2 = .11$. None of the interactions were significant:

- task x group: $F(1, 16) = 1.67$, $p = .22$, $\eta_p^2 = .06$
- task x ratio: $F(1, 16) = .24$, $p = .63$, $\eta_p^2 = .08$
- ratio x group: $F(1, 16) = .60$, $p = .45$, $\eta_p^2 = .01$
- task x ratio x group: $F(1, 16) = 2.88$, $p = .11$, $\eta_p^2 = .02$

Because we were interested in comparing the differences in performance between the two groups in the different tasks, individual ANOVAs were conducted for each experimental task (see Rousselle & Noël, 2007, for a similar procedure). In the HPC task, the ANOVA again showed a significant ratio effect: $F(1, 16) = 15.78$, $p = .001$, $\eta_p^2 = .46$. However, more interestingly, the group factor reached statistical significance, $F(1, 16) = 6.07$, $p = .026$, $\eta_p^2 = .27$, showing that the mean RT of the DHH children was 189 ms slower than that of their hearing peers. The interaction between group and ratio was not significant, $F(1, 16) = 2.26$, $p = .15$, $\eta_p^2 = .07$, indicating similar ratio effects for both groups.

In the LPC task, the ANOVA showed a ratio effect, $F(1, 16) = 22.95$, $p <.001$, $\eta_p^2 = .59$, but no group effects, $F(1, 16) = 0.27$, $p = .61$, $\eta_p^2 = .02$. The interaction between group and ratio was not significant, $F(1, 16) = .06$, $p = .81$, $\eta_p^2 < .01$.

Each group was analyzed for the effects of ratio and task. In the typically hearing children, the ANOVA revealed main effects of task, $F(1, 8) = 12.54$, $p = .008$, $\eta_p^2 = .61$, and ratio, $F(1, 8) = 11.63$, $p = .009$, $\eta_p^2 = .59$. The interaction between ratio and task was also significant, $F(1, 8) = 5.47$, $p = .048$, $\eta_p^2 = .40$, showing a longer-lasting ratio effect in the LPC task (255 ms) than in the HPC task (134 ms). Although the ratio effect was stronger in the LPC task, post hoc comparisons indicated that the ratio effect was, as expected, significant both in the LPC task, $p = .003$, and the HPC task, $p = .023$. However, for the performance of the DHH children, the ANOVA showed a main effect of ratio, $F(1, 8) = 14.76$, p = .005, $\eta_p^2 = .65$, although neither the task, $F(1, 8) = 1.68$, $p = .23$, $\eta_p^2 = .17$, nor the interaction between ratio and task, $F(1, 8) = .47$, $p = .51$, $\eta_p^2 = .06$, was significant.

The results of the ratio x task analysis in each group showed a stronger ratio effect for the LPC than for the HPC task in the typically hearing group. However, the size of the ratio effect was similar for both tasks in the DHH group, a finding that suggests that the DHH group did not take advantage of the perceptual cues when reaching a decision on the numerosity of the sets. Despite these subtle differences, it should be recalled that in the overall ANOVA the task x ratio x group interaction was not significant. We comment further on this aspect in the Discussion section.

Analysis of Accuracy

Table 3 shows the mean proportion of errors for each group in each ratio condition by task. As the distributions departed slightly from normality, data were arcsine transformed prior to analysis. The 2 x 2 x 2 ANOVA on the proportion of errors showed a main effect of task, $F(1, 16) = 16.86$, $p = .001$, $\eta_p^2 = .51$, and ratio, $F(1, 16) = 17.04$, $p = .001$, $\eta_p^2 = .52$. It should be noted that the group factor and the interaction between factors were far from statistical significant. (All p values were > .17.)

As in the RT analysis, each experimental task was analyzed separately. In the HPC task, a significant effect of ratio was found, $F(1, 16) = 14.25$, $p = .002$, $\eta_p^2 = .47$, although the group factor and the interaction group by ratio were not statistically significant: respectively, $F(1,16) = 0$, $F(1, 16) = 1.16$, $p = .3$, $\eta_p^2 = .07$. In the LPC task, a ratio effect was also found, $F(1, 16) = 9.06$, $p = .008$, $\eta_p^2 = .36$, but there were no group effects, $F(1, 16) = .016$, $p = .9$, $\eta_p^2 = .001$, or interactions between group and ratio, $F(1, 16) = 0.74$, $p = .4$, $\eta_p^2 = .04$.

Finally, in the hearing group, the ANOVA showed a main effect of task, $F(1, 8) = 9.89$, $p = .014$, $\eta_p^2 = .55$, and ratio, $F(1, 8) = 7.71$, $p = .024$, $\eta_p^2 = .49$, although the interaction between task and ratio was not significant, $F(1, 8) = 1.12$, $p = .32$, $\eta_p^2 = .12$. Similarly, in the DHH group, the ANOVA showed a main effect of task, $F(1, 8) = 7.48$, $p = .026$, $\eta_p^2 = .48$, and ratio, $F(1, 8) = 25$, $p = .001$, $\eta_p^2 = .76$. The interaction between task and ratio was not significant, $F(1, 8) = 1$, $p = .35$, $\eta_p^2 = .11$.

Discussion

The present study investigated whether the low level of mathematical achievement usually shown by DHH children may be attributed to a deficit in the functioning of the ANS. Two nonsymbolic comparison tasks were used to compare a group of DHH students and a group of typically hearing students who were matched by visual working memory, nonverbal IQ, and language comprehension. In the HPC task, perceptual features were manipulated to obtain a high correlation between the perceptual cues and numerosity, so that both types of information could be used to successfully perform the task. In contrast, the correlation between numerosity and perceptual cues was reduced in the LPC task, and therefore in this condition participants were forced to focus more on numerosity in order to do the task. It was predicted that a deficit in the ANS among the DHH participants would make performing the LPC task harder for them than for the hearing group.

The results of the analyses were clear: Overall, the behavior of the DHH participants was similar to that of the hearing children, a finding that suggests that there was no deficit in the ANS of the DHH sample. According to the literature (e.g., Inglis & Gilmore, 2014), accuracy is the most stable and reliable measure of the ANS, and in this sense the classic ratio effect was found in both groups; that is, achieving accuracy in the 1/2 ratio was easier than in the 2/3 ratio. Also as expected, the HPC task was easier than the LPC task. It is noteworthy that no group effects or interaction of group with the other factors were observed. The RT analysis produced similar results. The global ANOVA showed no group effects. Much as Rousselle and Noël (2007) found with the data they obtained, we found a main effect of task, a main effect of ratio, and a stronger ratio effect in the LPC condition than in the HPC condition. Although the task x group interaction was not significant, when response patterns were compared between groups the DHH group showed slower RTs in the HPC condition. Post hoc analyses also showed that the DHH group behaved similarly to the hearing group on both tasks. As we discuss below, we think that this difference does not indicate between-group differences in the integrity of the ANS.

The results show a discrepancy between performance in the addition fluency task, in which the DHH participants did worse than the hearing participants, and performance in the nonsymbolic comparison task, in which no differences were found between the two groups. These results contrast with the findings of two unpublished studies (Bull et al., 2010; Marschark et al, 2003) and, of more relevance, do not support the hypothesis of a defective number module, which was initially proposed by Butterworth (2005; see also Piazza et al., 2010) in regard to DD. According to this hypothesis, poor ANS acuity in DHH children should be responsible for their difficulties in the development of mathematical skills. Nevertheless, although the DHH children appeared to have difficulties with a fluency addition task, there was no evidence of differences between them and their hearing peers in two stick comparison tasks that were designed to evaluate the acuity of the ANS. In fact, our results are in line with those of previous studies that suggest that there are no differences between DHH participants and their hearing peers in nonsymbolic tasks (Arfé et al., 2011; Bull et al., 2006; Rodríguez-Santos et al., 2014; Zarfaty et al., 2004). Our results add to those of these studies, and the methodology we used overcame some of their limitations: (a) We used larger collections of dots (6–28, whereas previous studies used no more than 10), and (b) we avoided alternative explanations based on the use of perceptual cues by introducing the LPC condition, in which the correlation between perceptual and numerical cues was reduced.

We would also like to offer some comments on the effects of manipulating the perceptual cues in the comparison tasks. In general, performance was better in the HPC task than in the LPC task. In the HPC task, stick size and the distance between units were stable. The total surface area covered by the sticks and the length of the external contour occupied by the sticks differed between stick collections, and thus there was a strong correlation with numerosity. In contrast, in the LPC task, sticks of different sizes were used to match the total surface area covered by the sticks in each collection. In addition, the length of the external contour was also controlled. Only the density (i.e., the distance between sticks) inversely correlated with numerosity (see Figure 1).

The accuracy of DHH children and typically hearing children in each task was similar: For both groups, the LPC task was more difficult. Similarly, the typically hearing group had faster RTs in the HPC task than in the LPC task. Surprisingly, the DHH children were as slow in the HPC task as they were in the LPC task. Between-group comparisons in each task showed that the DHH students were as fast as the hearing students in the LPC task, but slower than their hearing peers in the HPC task. At first, it may seem contradictory that the performance of the DHH participants was similar to that of the hearing students in the difficult task but slower in the easier task, even though they achieved similar accuracy. Considering that in the LPC task only a reduced subset of perceptual cues are informative, and that therefore there is a greater demand for numerosity processing, the finding of similar performance by both groups provides the strongest evidence that the DHH children's nonsymbolic quantity processing was not impaired. The finding of a delay in responding to the HPC task among the DHH participants does not invalidate the finding of unimpaired nonsymbolic quantity processing. We do not have a clear explanation for the delayed response to the HPC task. It may be that the performance of DHH children in HPC tasks is influenced by other factors, such as instruction, as suggested by Pagliaro (2010), or a lack of cognitive flexibility, as suggested by Passig and Eden (2000) and Hall, Eigsti, Bortfeld, and Lillo-Martin (2016). For instance, difficulties in simultaneously managing different types of information (numerical and perceptual) may preclude DHH children from taking as much advantage of perceptual cues as their hearing peers do. For now, the present data suggest that any conclusion about this result must remain speculative. What is clear is that the delayed response to the HPC task was definitely not caused by a deficit in nonsymbolic representation; otherwise, a greater difference would have been observed in the task that demanded more numerosity-processing effort.

The data suggest that DHH students have no deficits in core numerical abilities, even though they show difficulties in mathematics achievement (in this case, addition fluency). Although an explanation for this finding is needed, at this point we can only speculate on this dissociation. By rejecting the existence of a deficit in the ANS, our data indirectly support the hypothesis of deficient access to numerical symbolic representations (Rousselle & Noël, 2007) or hypotheses that suggest that difficulties in mathematical development have a multifactorial origin (e.g., Kauffmann et al., 2013). It should be noted that these alternatives are not mutually exclusive. There is some support for the hypothesis that DHH students would have special difficulties with numerical symbolic representations. Thus, they would be unable to automatically activate quantity representation from Arabic or verbal notation despite having adequate numerosity processing ability. Rodríguez-Santos et al. (2014) found a delay in DHH children in accessing numerical representations from Arabic numerals, but not from fingers or dots. Similar difficulties using symbolic notation have been found in other studies with children and adults (e.g., Bull et al., 2005, 2011; Kritzer, 2009; Leybaert & Van Cutsem, 2002; Masataka, 2006).

It should also be noted that the low level of performance in symbolic numerical tasks does not appear to be explained by a low level of language. In the present study, as in others (e.g., Rodriguez et al., 2014), hearing and DHH groups were matched by language level. The symbolic deficit appears to be restricted to the Arabic and verbal notation systems, and it is here that experiential level may play a crucial role: As suggested by Kelly et al. (2003), DHH children seem to experience reduced exposure to numerical concepts. If this is the case, the numerical skills of DHH children would be improved by educational programs that increased their exposure to numerical symbolic notations. However, this hypothesis does not apply to the difficulties DHH children show with higher levels of numerical processing, such as complex arithmetic or word problems in which the language demands are high (Hyde, Zevenbergen, & Power, 2003; Kelly et al., 2003; Pagliaro & Ansell, 2012). Further research would be needed to test this hypothesis.

Limitations of the Study

It would be unfair to finish this discussion without recognizing the limitations of the present study. A small number of participants in a study can affect the detection of true differences between groups as well as the generalizability of the results (Cohen, 1977). The present study had nine participants per group, which increased the risk of a type II error (i.e., failing to reject the null hypothesis when it is false). However, group effects, or interactions with the factor group involved, were always far from statistical significance (*p* values >.1). In fact, a detailed analysis of the 2/3 ratio in the LPC condition, which demands greater numerosity processing, showed that the DHH group even had a numerical advantage (see Table 3). Hence, although it cannot be completely discounted, it is unlikely that the absence of differences is due to the small number of participants per group. Regarding the generalizability of our results, it should be taken into account that our DHH sample was not typical. The sample was recruited at a center for DHH students that employs bimodal communication and uses a Cued Speech system to teach oral language in a very systematic way. This center is unique in Spain, and hence it is unlikely that the sample was representative of the entire DHH population. However, this particular concern is applicable to most of the research conducted with DHH students.

Conclusion

It is thought that DHH children have fewer opportunities to experience mathematical concepts as a consequence of their auditory difficulties (Kelly et al., 2003). Some evidence also suggests that at school they are offered less diverse strategies to address problem solving. Although this may affect the development of their ANS, our results do not support this view. The DHH students were as able as hearing participants to process nonsymbolic numerosities. Difficulties in more complex numerical tasks, such as addition, were not associated with the processing of nonsymbolic magnitudes.

The origin of the deficits should be encountered at higher levels. As suggested by recent studies, it can be plausibly hypothesized that a deficit exists in matching symbolic representations (Arabic and verbal numbers) with their corresponding nonsymbolic representations (in line with the access deficit hypothesis). However, further research is needed to shed light on this matter.

A Note on Funding

The research for the present article was supported by a grant from the Consejería de Economía, Innovación, Ciencia y Empleo de la Junta de Andalucía (Spain), Grant No. P07-SEJ-03220, and by a grant from the Spanish Ministry of Economy and Competitiveness (MINECO), Grant No. PSI-2012-38423.

Notes on Terminology

1. We employ the term *language* in a broad sense, that is, as including oral as well as signed languages. We assume that differences in mastering a language have an impact on numerical skills, especially in symbolic numerical processing (Göbel et al., 2014).

2. In general, we consider communication to be bimodal when there is simultaneous use of speech and signs; that is, communication requires the use of an oral-auditory modality together with a visual-gestural modality. Though the message is simultaneously expressed in two modalities, the base language is the oral language (i.e., the one that marks the order of the phrase and that determines the syntax of the productions).

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