Adaptive selective relaying in cooperative free-space optical systems over atmospheric turbulence and misalignment fading channels

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Abstract: In this paper, a novel adaptive cooperative protocol with multiple relays using detect-and-forward (DF) over atmospheric turbulence channels with pointing errors is proposed. The adaptive DF cooperative protocol here analyzed is based on the selection of the optical path, source-destination or different source-relay links, with a greater value of fading gain or irradiance, maintaining a high diversity order. Closed-form asymptotic bit error-rate (BER) expressions are obtained for a cooperative free-space optical (FSO) communication system with N_r relays, when the irradiance of the transmitted optical beam is susceptible to either a wide range of turbulence conditions, following a gamma-gamma distribution of parameters α and β, or pointing errors, following a misalignment fading model where the effect of beam width, detector size and jitter variance is considered. A greater robustness for different link distances and pointing errors is corroborated by the obtained results if compared with similar cooperative schemes or equivalent multiple-input multiple-output (MIMO) systems. Simulation results are further demonstrated to confirm the accuracy and usefulness of the derived results.

OCIS codes: (010.1330) Atmospheric turbulence; (060.2605) Free-space optical communication; (060.4510) Optical communications.

References and links

1. V. W. S. Chan, "Free-space optical communications," J. Lightwave Technol. **24**(12), 4750–4762 (2006).
2. L. Andrews, R. Phillips, and C. Hopen, *Laser Beam Scintillation with Applications* (Bellingham, WA: SPIE Press, 2001).
3. M. Safari and M. Uysal, "Relay-assisted free-space optical communication," IEEE Trans. Wireless Commun. **7**(12), 5441–5449 (2008).
4. M. Karimi and M. Nasiri-Kenari, "BER analysis of cooperative systems in free-space optical networks," J. Lightwave Technol. **27**(24), 5639 –5647 (2009).
5. M. Karimi and M. Nasiri-Kenari, "Outage analysis of relay-assisted free-space optical communications," IET Communications **4**(12), 1423 –1432 (2010).
6. Y. Celik and N. Odabasioglu, "On relay selection for cooperative free-space optical communication," in *Networks and Optical Communications (NOC), 2012 17th European Conference on*, pp. 1–5 (IEEE, 2012).
7. A. Garcia-Zambrana, C. Castillo-Vazquez, B. Castillo-Vazquez, and R. Boluda-Ruiz, "Bit detect and forward relaying for FSO links using equal gain combining over gamma-gamma atmospheric turbulence channels with pointing errors," Opt. Express **20**(15), 16394–16409 (2012).

8. S. I. Hussain, M. M. Abdallah, and K. A. Qaraqe, "Power optimization and k th order selective relaying in free space optical networks," in *GCC Conference and Exhibition (GCC), 2013 7th IEEE*, pp. 330–333 (IEEE, 2013).
9. C. Abou-Rjeily, "Performance Analysis of Selective Relaying in Cooperative Free-Space Optical Systems," J. Lightwave Technol. **31**(18), 2965–2973 (2013).
10. C. Abou-Rjeily, "Achievable Diversity Orders of Decode-and-Forward Cooperative Protocols over Gamma-Gamma Fading FSO Links," IEEE Trans. Commun. **61**(9), 3919–3930 (2013).
11. N. D. Chatzidiamantis, D. S. Michalopoulos, E. E. Kriezis, G. K. Karagiannidis, and R. Schober, "Relay selection protocols for relay-assisted free-space optical systems," J. Opt. Commun. Netw. **5**(1), 92–103 (2013).
12. M. R. Bhatnagar, "Average BER analysis of relay selection based decode-and-forward cooperative communication over Gamma-Gamma fading FSO links," in *IEEE International Conference on Communications (ICC)*, pp. 3142–3147 (2013).
13. M. A. Kashani and M. Uysal, "Outage performance and diversity gain analysis of free-space optical multi-hop parallel relaying," J. Opt. Commun. Netw. **5**(8), 901–909 (2013).
14. L. Yang, X. Gao, and M.-S. Alouini, "Performance Analysis of Free-Space Optical Communication Systems with Multiuser Diversity Over Atmospheric Turbulence Channels," IEEE Photonics J. **6**(2), 7901217 (2014).
15. I. I. Kim, B. McArthur, and E. J. Korevaar, "Comparison of laser beam propagation at 785 nm and 1550 nm in fog and haze for optical wireless communications," in *Information Technologies 2000*, pp. 26–37 (International Society for Optics and Photonics, 2001).
16. D. K. Borah and D. G. Voelz, "Pointing error effects on free-space optical communication links in the presence of atmospheric turbulence," J. Lightwave Technol. **27**(18), 3965–3973 (2009).
17. A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," J. Lightwave Technol. **25**(7), 1702–1710 (2007).
18. H. G. Sandalidis, T. A. Tsiftsis, and G. K. Karagiannidis, "Optical wireless communications with heterodyne detection over turbulence channels with pointing errors," J. Lightwave Technol. **27**(20), 4440–4445 (2009).
19. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. (Academic Press Inc., 2007).
20. M. A. Al-Habash, L. C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," Opt. Eng. **40**, 8 (2001).
21. N. Wang and J. Cheng, "Moment-based estimation for the shape parameters of the gamma-gamma atmospheric turbulence model." Opt. Express **18**(12), 12824–12831 (2010).
22. A. García-Zambrana, B. Castillo-Vázquez, and C. Castillo-Vázquez, "Asymptotic error-rate analysis of FSO links using transmit laser selection over gamma-gamma atmospheric turbulence channels with pointing errors," Opt. Express **20**(3), 2096–2109 (2012).

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1. Introduction

Free-space optical (FSO) communications can provide high-speed links for a variety of applications. The most special characteristics are unlimited bandwidth, unlicensed spectrum, excellent security, and low cost [1]. The use of FSO communication systems is being specially interesting to solve the *last mile* problem when fiber-optic links are not practical, as well as a supplement to radio-frequency (RF) links. Among the most important disadvantages are the atmospheric propagation factors, such as haze, fog, rain and snow. However, the most serious problem is the atmospheric turbulence, which produces fluctuations in the irradiance of the transmitted optical beam, as a result of variations in the refractive index along the link [2]. Additionally, the high directivity of the transmitted beam in FSO systems, or building sway can produce an unsuitable alignment between transmitter and receiver and, hence, a greater deterioration in performance. To solve these inconveniences the adoption of forward error correcting codes as well as spatial diversity based on multiple-input multiple-output (MIMO) configurations is proposed. An alternative approach to improve the performance in this turbulent FSO scenario is based on the employment of cooperative communications in order to overcome some limitations of MIMO structures. Cooperative transmission can significantly enhance the performance, increasing diversity by using the transceivers available at the other nodes of the network.

Much research in recent years has focused on this technique in the context of FSO communications [3–14]. In [3] an artificial broadcasting through the use of multiple transmitter apertures directed to relay nodes is proposed as a parallel relaying transmission scheme as well as a serial transmission, evaluating the outage probability when amplify-and-forward (AF) and decode-and-forward (DF) relaying are considered. In [4, 5] a 3-way FSO system is proposed to implement a cooperative protocol in order to improve spatial diversity without much increase in hardware, being evaluated the error-rate performance, using the photon-count method, as well as the outage performance. In particular, an adaptive cooperative protocol is presented in [4] wherein the source node participates in cooperation only if the source-relay link is in an acceptable quality. In [6] two relay selection methods for a cooperative FSO system with DF relaying are introduced, being evaluated the outage probability by using the photon-count method with a log-normal turbulence-induced fading channel model. In [7], following the bit-detect-and-forward (BDF) cooperative protocol presented in [4], the analysis is extended over gamma-gamma and misalignment fading channels assuming equal gain combining (EGC) in reception. In [8] a relay selection protocol is investigated where the relay is operating in AF mode under log-normal fading, deriving outage probability of the system. In [10], two cooperative protocols based on relay selection for any number of relays are analyzed, proposing an alternative metric of selection obtaining high diversity order gain over gamma-gamma fading channels without pointing errors, by using the photon-count method in the presence and absence of background radiation. In [11] several transmission protocols are investigated for a cooperative FSO system over gamma-gamma fading channels, proposing alternative protocols to the all-active relaying scheme, which activate only a single relay in each transmission slot. In [12], DF based on FSO cooperative systems over gamma-gamma fading channels with relay selection are analyzed, wherein the source selects a relay on the basis of highest instantaneous signal-to-noise ratio (SNR) of the source-relay links. In [13], a cooperative FSO system is analyzed in which serial and parallel relaying are deployed together, being obtained the outage probability over log-normal fading channels. In [14], a cooperative FSO communication system is analyzed based on users selection, where the best user is selected, being evaluated the outage probability and bit error-rate (BER) over log-normal and gamma-gamma fading channels.

In this paper, a novel adaptive cooperative protocol with multiple relays is presented, wherein the relays are operating in DF mode over atmospheric turbulence channels with pointing errors. The adaptive-detect-and-forward (ADF) relaying scheme here proposed is based on the selection of the optical path, source-destination or different source-relay links, with a greater value of fading gain or irradiance, maintaining a high diversity order. Closed-form asymptotic BER expressions are obtained for a cooperative FSO communication system with N_r relays, when the irradiance of the transmitted optical beam is susceptible to moderate-to-strong turbulence conditions, following a gamma-gamma distribution of parameters α and β, or pointing errors, following a misalignment fading model where the effect of beam width, detector size and jitter variance is considered. The diversity order gain analysis is performed as a function of the number of relays and different atmospheric turbulence conditions. A greater robustness for different link distances and pointing errors is corroborated by the obtained results if compared with similar cooperative FSO schemes or equivalent MIMO systems with multiple transmitters. Simulation results are further demonstrated to confirm the accuracy and usefulness of the derived results.

2. System and channel model

The system model under study is shown in Fig. 1. We consider a cooperative FSO system with multiple relays, consisting of one source node S, N_r relays denoted by R_k for $k = \{1, 2, \ldots, N_r\}$, and one destination node D. In this study, different relay-destination link distances within the range d'_{RD} to d_{RD} ($d'_{RD} < d_{RD}$) are considered in order to perform a more realistic analysis in the context of FSO cooperative systems, assuming that these relays are located in an area similar to an annullus. In particular, we adopt laser sources intensity-modulated and ideal non-coherent (direct-detection) receivers. The ADF relaying scheme with multiple relays here proposed selects between direct-transmission (DT) or BDF cooperative protocol presented in [7] on the

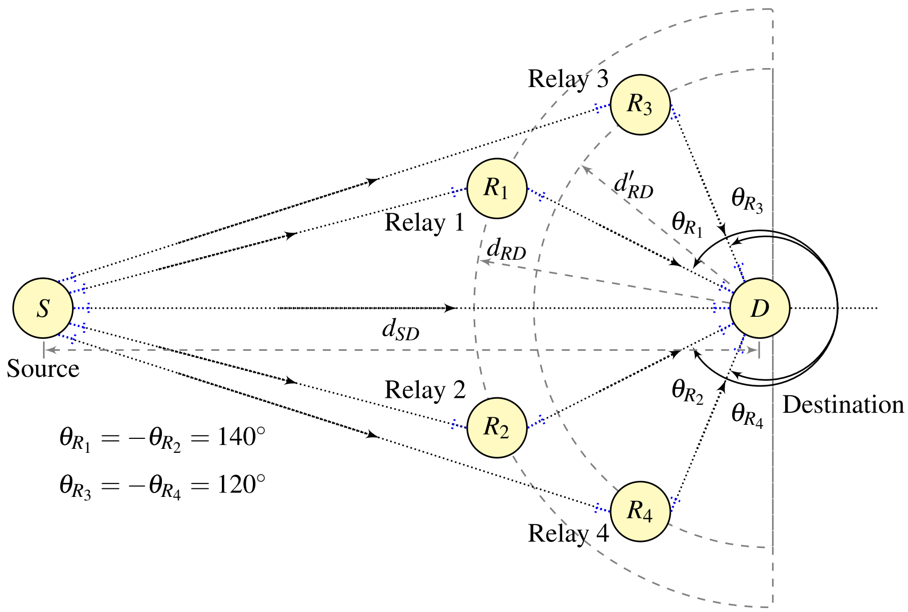

Fig. 1. Block diagram of the cooperative FSO system under study, where d_{SD} is the source-destination link distance, R_k are the relays nodes for $k = \{1, 2, \ldots, N_r\}$, and (d_{R_kD}, ϕ_{R_k}) represents the relay location using polar coordinates.

basis of the value of the fading gain. When the irradiance of the source-destination link (I_{SD}) is greater than the irradiances corresponding to the source-relay links (I_{SR_k} for $k = \{1, 2, \ldots, N_r\}$), the FSO communication system is only based on the direct transmission to the destination node, obviating the cooperative mode. On contrary, the source node S performs cooperation using the relay R_k if the irradiance of the S-R_k link is greater than the corresponding irradiance of the S-D link and the irradiances S-R_i for $i \neq k$, i.e. the remaining relays. This metric is able to select the best relay node with better possible conditions in order to detect correctly the received information from the source node, since a decode-and-forward relaying scheme is being considered. In contrast to [4], wherein the cooperative strategy is not considered when the source-relay links quality in terms of $I^2_{SR_k}$ falls below a certain threshold, it is here assumed that cooperative communications are not established when the irradiance of S-D link takes the greater value if compared to the irradiances of the source-relay links. It must be noted that the selection criterion is here based on the value of the irradiance instead of the square magnitude, as previously commented when comparing to a certain threshold or when selecting the maximum value of the instantaneous received SNR at the relay, as usually is assumed in the literature [6, 8, 11, 12, 14]. It is taking into account that CSI is known not only at the receiver but also at the transmitter (CSIT). The knowledge of CSIT is feasible for FSO channels given that scintillation is a slow time varying process relative to the large symbol rate. In this sense, the receiver always knows if the cooperative protocol is being used. The ADF cooperative protocol works in two phases or transmission frames, as shown in Table 1. It can be observed that, for example, no cooperative transmission is being used in $slot_2$ and $slot_5$ since the irradiance of the S-D link is greater than the corresponding irradiances of the S-R_k links in these particular slots. It must be noted that one transmission overlapped implies that no rate reduction is applied and, hence, the same information rate can be considered at the destination node D compared to the direct transmission link without using any cooperative strategy. Here, it is assumed that all the bits detected at the relay node selected by the source node are always resended with the new power to the destination node D regardless of these bits detected correctly or incorrectly, unlike [10].

Table 1. ADF cooperative protocol with multiple relays when the message $s = \{s_1, s_2, \ldots, s_n\}$ is transmitted, being $R_{\max}$ the relay selected by the source node.

Link	$slot_1$	$slot_2$	$slot_3$	$slot_4$	$slot_5$	$slot_6$	$slot_7$	**...**	$slot_n$	$slot_{n+1}$
S-D	s_1	s_2	s_3	s_4	s_5	s_6	s_7	$\ldots$	s_n	–
S-$R_{\mathbf{max}}$	s_1	–	s_3	s_4	–	s_6	s_7	$\ldots$	s_n	–
$R_{\mathbf{max}}$-D	–	s_1^*	–	s_3^*	s_4^*	–	s_6^*	$\ldots$	s_{n-1}^*	s_n^*

For each link of this cooperative FSO communications system, the instantaneous current $y_m(t)$ in the receiving photodetector corresponding to the information signal transmitted from each laser can be written as

$$y_m(t) = \eta\, i_m(t) x_m(t) + z_m(t), \tag{1}$$

where η is the detector responsivity, assumed hereinafter to be the unity, $X \triangleq x_m(t)$ represents the optical power supplied by the source, $I_m \triangleq i_m(t)$ the equivalent real-value fading gain (irradiance) through the optical channel between the laser and the receive aperture. $Z_m \triangleq z_m(t)$ is assumed to include any front-end receiver thermal noise as well as shot noise caused by ambient light much stronger than the desired signal at the detector. It can usually be modeled to high accuracy as AWGN with zero mean and variance $\sigma^2 = N_0/2$, i.e. $Z_m \sim N(0, N_0/2)$, independent of the on/off state of the received bit. We use X, Y_m, I_m and Z_m to denote random variables and $x_m(t)$, $y_m(t)$, $i_m(t)$ and $z_m(t)$ their corresponding realizations. The irradiance is considered to be a product of three factors i.e., $I_m = L_m I_m^{(a)} I_m^{(p)}$ where L_m is the deterministic propagation loss, $I_m^{(a)}$ is the attenuation due to atmospheric turbulence and $I_m^{(p)}$ the attenuation due to geometric spread and pointing errors. L_m is determined by the exponential Beers-Lambert law as $L_m = e^{-\Phi d}$, where d is the link distance and Φ is the atmospheric attenuation coefficient. It is given by $\Phi = (3.91/V(km))\,(\lambda(nm)/550)^{-q}$ where V is the visibility in kilometers, λ is the wavelength in nanometers and q is the size distribution of the scattering particles, being $q = 1.3$ for average visibility (6 km $< V <$ 50 km), and $q = 0.16V + 0.34$ for haze visibility (1 km $< V <$ 6 km) [15]. Although the effects of turbulence and pointing are not strictly independent, for smaller jitter values they can be approximated as independent [16]. To consider a wide range of turbulence conditions, the gamma-gamma turbulence model proposed in [2] is here assumed. Regarding to the impact of pointing errors, we use the general model of misalignment fading given in [17] by Farid and Hranilovic, wherein the effect of beam width, detector size and jitter variance is considered. A closed-form expression of the combined probability density function (PDF) of I_m was derived in [18] as

$$f_{I_m}(i) = \frac{\alpha_m \beta_m \varphi_m^2}{A_0 L_m \Gamma(\alpha_m)\Gamma(\beta_m)} G_{1,3}^{3,0}\left(\frac{\alpha_m \beta_m}{A_0 L_m} i \,\middle|\, \begin{matrix} \varphi_m^2 \\ \varphi_m^2 - 1, \alpha_m - 1, \beta_m - 1 \end{matrix} \right), \quad i \geq 0 \tag{2}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function [19, eqn. (9.301)] and $\Gamma(\cdot)$ is the well-known Gamma function. Assuming plane wave propagation, α and β can be directly linked to physical param-

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eters through the following expresions [20]:

$$\alpha=\left[\exp\left(0.49\sigma_R^2/(1+1.11\sigma_R^{12/5})^{7/6}\right)-1\right]^{-1} \tag{3a}$$

$$\beta=\left[\exp\left(0.51\sigma_R^2/(1+0.69\sigma_R^{12/5})^{5/6}\right)-1\right]^{-1} \tag{3b}$$

where $\sigma_R^2=1.23C_n^2\kappa^{7/6}d^{11/6}$ is the Rytov variance, which is a measure of optical turbulence strength. Here, $\kappa=2\pi/\lambda$ is the optical wave number and d is the link distance in meters. C_n^2 stands for the altitude-dependent index of the refractive structure parameter and varies from 10^{-13} $m^{-2/3}$ for strong turbulence to 10^{-17} $m^{-2/3}$ for weak turbulence [2]. It must be emphasized that parameters α and β cannot be arbitrarily chosen in FSO applications, being related through the Rytov variance. It can be shown that the relationship $\alpha>\beta$ always holds, and the parameter β is lower bounded above 1 as the Rytov variance approaches ∞ [21]. In relation to the impact of pointing errors [17], assuming a Gaussian spatial intensity profile of beam waist radius, ω_z, on the receiver plane at distance z from the transmitter and a circular receive aperture of radius r, $\varphi=\omega_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver, $\omega_{z_{eq}}^2=\omega_z^2\sqrt{\pi}\mathrm{erf}(v)/2v\exp(-v^2)$, $v=\sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0=[\mathrm{erf}(v)]^2$ and $\mathrm{erf}(\cdot)$ is the error function [19, eqn. (8.250)].

Nonetheless, the PDF in Eq. (2) appears to be cumbersome to use in order to obtain simple closed-form expressions in the analysis of FSO communication systems. To overcome this inconvenience, the PDF is approximated by a single polynomial term as $f_{I_m}(i)\approx a_m i^{b_m-1}$, based on the fact that the asymptotic behavior of the system performance is dominated by the behavior of the PDF near the origin [7]. Different expressions for $f_{I_m}(i)$, depending on the relation between the values of φ^2 and β, can be written as

$$f_{I_m}(i)\approx a_m i^{b_m-1}=\frac{\varphi_m^2(\alpha_m\beta_m)^{\beta_m}\Gamma(\alpha_m-\beta_m)}{(A_0L_m)^{\beta_m}\Gamma(\alpha_m)\Gamma(\beta_m)\,(\varphi_m^2-\beta_m)}i^{\beta_m-1}, \qquad \varphi_m^2>\beta_m \tag{4a}$$

$$f_{I_m}(i)\approx a_m i^{b_m-1}=\frac{\varphi_m^2(\alpha_m\beta_m)^{\varphi_m^2}\Gamma\left(\alpha_m-\varphi_m^2\right)\Gamma\left(\beta_m-\varphi_m^2\right)}{(A_0L_m)^{\varphi_m^2}\Gamma(\alpha_m)\Gamma(\beta_m)}i^{\varphi_m^2-1}, \quad \varphi_m^2<\beta_m \tag{4b}$$

In the following section, the fading coefficient I_m for the paths S-D, S-R_k and R_k-D is indicated by I_{SD}, I_{SR_k} and I_{R_kD}, respectively, for $k=\{1,2,\ldots,N_r\}$. The subscript k is used to represent the different relays, which can be selected by the source node in each transmission frame. Here, we assume that all coefficients are independent statistically.

3. Error-rate performance analysis

In this section, the ADF cooperative protocol with multiple relays is analyzed in order to obtain asymptotic expressions for the bit error probability at high SNR, showing that the asymptotic performance of this metric as a function of the average SNR is characterized by two parameters: the diversity and coding gains. In addition to the performance evaluation of the BER corresponding to the ADF cooperative protocol here proposed, we also consider the performance analysis for the direct transmission (non-cooperative S-D link) to establish the baseline performance. The average optical power transmitted from each node is P_{opt}, being adopted an on-off keying (OOK) signaling based on a constellation of two equiprobable points in a one-dimensional space with an Euclidean distance of $d_E=2P_{\mathrm{opt}}\sqrt{T_b\xi}$, where the parameter T_b is the bit period, and ξ represents the square of the increment in Euclidean distance due to the use of a pulse shape of high peak-to-average optical power ratio (PAOPR) [7]. According to Eq. (1),

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the statistical channel model corresponding to the source-destination link without cooperative communication can be written as

$$Y_{SD} = XI_{SD} + Z_{SD}, \quad X \in \{0, d_E\}, \quad Z_{SD} \sim N(0, N_0/2). \tag{5}$$

Assuming channel side information at the receiver, the conditional BER at the destination node is given by

$$P_b^{SD}(E|I_{SD}) = Q\left(\sqrt{d_E^2 i^2/2N_0}\right) = Q\left(\sqrt{2\gamma\xi}\, i\right), \tag{6}$$

where $Q(\cdot)$ is the Gaussian Q-function and $\gamma = P_{\rm opt}^2 T_b/N_0$ represents the received electrical SNR in absence of turbulence. Hence, the average BER, $P_b^{SD}(E)$, can be obtained by averaging $P_b^{SD}(E|I_{SD})$ over the PDF as follows

$$P_b^{SD}(E) = \int_0^\infty Q\left(\sqrt{2\gamma\xi}\, i\right) f_{I_{SD}}(i) di. \tag{7}$$

To evaluate the integral in Eq. (7), we can use that the Q-function is related to the complementary error function $\mathrm{erfc}(\cdot)$ by $\mathrm{erfc}(x) = 2Q(\sqrt{2}x)$ [19, eqn. (6.287)] and the fact that $\int_0^\infty \mathrm{erfc}(x)x^{a-1}dx = \Gamma((1+a)/2)/(\pi^{1/2}a)$ [19, eqn. (6.281)], obtaining the corresponding closed-form asymptotic solution for the BER as can be seen in

$$P_b^{SD}(E) \doteq \frac{a_{SD}\Gamma((b_{SD}+1)/2)}{2b_{SD}\sqrt{\pi}} (\gamma\xi)^{-b_{SD}/2}, \tag{8}$$

where the value of the parameters a_{SD} and b_{SD} depends on the relation between φ_{SD}^2 and β_{SD}, as shown in Eq. (4), corroborating that the diversity order corresponding to the source-destination link is independent of the pointing errors when $\varphi_{SD}^2 > \beta_{SD}$. The statistical channel model corresponding to the ADF cooperative protocol using N_r relays can be written as

$$Y_{ADF}^0 = \frac{1}{2}XI_{SD} + Z_{SD} + X^* I_{R_{\max}D} + Z_{R_{\max}D}, \quad I_{SR_{\max}} > I_{SD} \tag{9a}$$

$$Y_{ADF}^1 = XI_{SD} + Z_{SD}, \quad I_{SR_{\max}} < I_{SD} \tag{9b}$$

being $R_{\max}$ the relay node where the value of the fading gain corresponding to the S-$R_{\max}$ link is maximum, denoted as $I_{SR_{\max}}$, i.e. $I_{SR_{\max}} = \max\{I_{SR_1}, I_{SR_2}, \ldots, I_{SR_N}\}$, $X \in \{0, d_E\}$ and $Z_{SD}, Z_{R_{\max}D} \sim N(0, N_0/2)$. X^* represents the random variable corresponding to the information detected at the node $R_{\max}$ and, hence, $X^* = X$ when the bit has been detected correctly, and $X^* = d_E - X$ when the bit has been detected incorrectly at $R_{\max}$. The Eq. (9a) is the statistical channel model corresponding to the BDF cooperative protocol analyzed in [7] and, hence, depending on the fact that the bit from the relay S-$R_{\max}$-D is detected correctly or incorrectly, it can be expressed as

$$Y_{ADF}^0 = (1/2)X\left(I_{SD} + 2I_{R_{Max}D}\right) + Z_{SD} + Z_{R_{Max}D}, \quad X^* = X \tag{10a}$$

$$Y_{ADF}^0 = (1/2)X\left(I_{SD} - 2I_{R_{Max}D}\right) + d_E \cdot I_{R_{Max}D} + Z_{SD} + Z_{R_{Max}D}, \quad X^* = d_E - X \tag{10b}$$

The Eq. (9b) is the statistical channel corresponding to the direct transmission without cooperative communication. Assuming channel side information at the receiver and transmitter, the conditional BER at the node D, $P_b^{ADF}(E|I_{SD}, I_{SR_k}, I_{R_kD})$ for $k = \{1, 2, \ldots, N_r\}$, can be written as

$$\begin{aligned} &P_b^{ADF}(E|I_{SD}, I_{SR_k}, I_{R_kD}) \\ &= \sum_{k=1}^{N_r} P_b^{BDF_k}(E|I_{SD}, I_{SR_k}, I_{R_kD}) F_{I_{SD}}(I_{SR_k}) \prod_{\substack{j=1\\ j\neq k}}^{N_r} F_{I_{SR_j}}(I_{SR_k}) + P_b^{SD}(E|I_{SD}) \prod_{j=1}^{N_r} F_{I_{SR_j}}(I_{SD}), \end{aligned} \tag{11}$$

being $F_{I_m}(i)$ the cumulative density function (CDF) of the random variable I_m, which is given by $F_{I_m}(i) = \text{Prob}(I_m \leq i)$. Knowing the fact that the irradiances are statistically independent, the probability corresponding to $I_{SD} > I_{SR_j}$ is computed by using $F_{I_{SR_j}}(I_{SD})$. In the same way, the probability corresponding to $I_{SR_j} > I_{SD}$ can be computed by using $F_{I_{SD}}(I_{SR_j})$. For the sake of simplicity, in spite of the fact that this CDF can be expressed in terms of generalized hypergeometric functions, an approximated expression by a single polynomial term as $F_{I_m}(i) \approx (a_m/b_m)i^{b_m}$ is here assumed, as can easily be deduced from Eq. (4). $P_b^{BDF_k}(E|I_{SD}, I_{SR_k}, I_{R_kD})$ is the conditional BER corresponding at the node D when the relay node R_k is selected by the source node, as follows

$$\begin{aligned}
&P_b^{BDF_k}(E|I_{SD}, I_{SR_k}, I_{R_kD}) \\
&= P_b^{BDF_k^0}(E|I_{SD}, I_{R_kD})(1 - P_b^{SR_k}(E|I_{SR_k})) + P_b^{BDF_k^1}(E|I_{SD}, I_{R_kD})P_b^{SR_k}(E|I_{SR_k}) \\
&= Q\left(\sqrt{(\gamma/4)\xi}(i_{SD} + 2i_{R_kD})\right)\left(1 - Q\left(\sqrt{(\gamma/2)\xi}\, i_{SR_k}\right)\right) \\
&+ Q\left(\sqrt{(\gamma/4)\xi}(i_{SD} - 2i_{R_kD})\right) Q\left(\sqrt{(\gamma/2)\xi}\, i_{SR_k}\right).
\end{aligned} \tag{12}$$

The division by 2 in $P_b^{SR_k}(E|I_{SR_k})$ is considered so as to maintain the average optical power in the air at a constant level of P_{opt}, being transmitted by each laser an average optical power $P_{opt}/2$ in the first phase when only relay R_k is selected by the source node. Hence, the average BER, $P_b^{ADF}(E)$, can be obtained by averaging $P_b^{ADF}(E|I_{SD}, I_{SR_k}, I_{R_kD})$ over the PDFs as follows

$$\begin{aligned}
P_b^{ADF}(E) &= \sum_{k=1}^{N_r}\left(P_b^{BDF_k^0}(E)P_b^{SR_{kADF}^0}(E) + P_b^{BDF_k^1}(E)P_b^{SR_{kADF}^1}(E)\right) + P_b^{SD_{ADF}}(E) \\
&= \sum_{k=1}^{N_r} P_b^{BDF_k^0}(E)\int_0^\infty [1 - P_b^{SR_k}(E|I_{SR_k})]F_{I_{SD}}(i)\prod_{\substack{j=1\\ j\neq k}}^{N_r} F_{I_{SR_j}}(i) f_{I_{SR_k}}(i)di. \\
&+ \sum_{k=1}^{N_r} P_b^{BDF_k^1}(E)\int_0^\infty P_b^{SR_k}(E|I_{SR_k})F_{I_{SD}}(i)\prod_{\substack{j=1\\ j\neq k}}^{N_r} F_{I_{SR_j}}(i) f_{I_{SR_k}}(i)di. \\
&+ \int_0^\infty P_b^{SD}(E|I_{SD})\prod_{j=1}^{N_r} F_{I_{SR_j}}(i) f_{I_{SD}}(i)di.
\end{aligned} \tag{13}$$

Next, the following generic expression is computed for the BER corresponding to the S-R_k and S-D links as follows

$$\int_0^\infty Q\left(\sqrt{c\gamma\xi}\, i\right)\prod_{\substack{n=1\\ n\neq m}}^{N_r} F_{I_n}(i) f_{I_m}(i)di, \quad c \in \mathbb{R}^+ \tag{14}$$

In particular, the parameter c is set to 2 in Eq. (14) for the BER corresponding to the S-D link. This expression, corresponding to a direct transmission without cooperation, i.e. $P_b^{SD_{ADF}}(E)$, is given by

$$P_b^{SD_{ADF}}(E) = \int_0^\infty P_b^{SD}(E|I_{SD})\prod_{j=1}^{N_r} F_{I_{SR_j}}(i) f_{I_{SD}}(i)di. \tag{15}$$

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In a similar way, when the parameter c is set to 1/2 in Eq. (14), the BER corresponding to the S-R_k link, i.e. $P_b^{SR_{k}{}^{1}_{ADF}}(E)$, is computed as follows

$$P_b^{SR_{k}{}^{1}_{ADF}}(E) = \int_0^\infty P_b^{SR_k}(E|I_{SR_k}) F_{I_{SD}}(i) \prod_{\substack{j=1 \\ j\neq k}}^{N_r} F_{I_{SR_j}}(i) f_{I_{SR_k}}(i) di. \tag{16}$$

Evaluating the Eq. (15) and Eq. (16) as in Eq. (7), we can obtain the corresponding closed-form asymptotic solution for the BER as can be seen in

$$P_b^{SD_{ADF}}(E) \doteq \frac{a_{SD}\prod_{i=1}^{N_r} a_{SR_i}\Gamma\left(\left(1+b_{SD}+\sum_{i=1}^{N_r} b_{SR_i}\right)/2\right)(\gamma\xi)^{-\left(b_{SD}+\sum_{i=1}^{N_r} b_{SR_i}\right)/2}}{\prod_{i=1}^{N_r} b_{SR_i} 2\sqrt{\pi}\left(b_{SD}+\sum_{i=1}^{N_r} b_{SR_i}\right)} \tag{17}$$

and

$$P_b^{SR_{k}{}^{1}_{ADF}}(E) \doteq \frac{b_{SR_k}}{b_{SD}} 2^{\left(b_{SD}+\sum_{i=1}^{N_r} b_{SR_i}\right)} \cdot P_b^{SD_{ADF}}(E), \tag{18}$$

respectively. Finally, the probability when the bit is detected correctly at the node R_k, i.e. $P_b^{SR_{k}{}^{0}_{ADF}}(E)$, is computed as

$$P_b^{SR_{k}{}^{0}_{ADF}}(E) = \int_0^\infty [1 - P_b^{SR_k}(E|I_{SR_k})] F_{I_{SD}}(i) \prod_{\substack{j=1 \\ j\neq k}}^{N_r} F_{I_{SR_j}}(i) f_{I_{SR_k}}(i) di. \tag{19}$$

Here, we can use that de Gaussian Q-function tends to 0 as $\gamma \to \infty$, simplifying the integral in Eq. (19) as follows

$$P_b^{SR_{k}{}^{0}_{ADF}}(E) \doteq \int_0^\infty \prod_{\substack{j=1 \\ j\neq k}}^{N_r} F_{I_{SR_j}}(i) F_{I_{SD}}(i) f_{I_{SR_k}}(i) di. \tag{20}$$

It can be noted that the asymptotic behavior of $P_b^{SR_{k}{}^{0}_{ADF}}(E)$ is independent of the SNR γ, resulting in a positive value that is upper bounded by 1. Here, the Eq. (20) was obtained by using the Monte Carlo integration, being analytically intractable. The corresponding closed-form asymptotic solution for the BER corresponding to the BDF relaying scheme was derived in [7, Eq. (18) and Eq. (21)], i.e. $P_b^{BDF_k^0}(E)$ and $P_b^{BDF_k^1}(E)$, respectively. Using the notation here assumed, these expressions are here rewritten as

$$P_b^{BDF_k^0}(E) \doteq \frac{a_{SD} a_{R_kD} \Gamma(b_{SD}) \Gamma(b_{R_kD}) (\gamma\xi)^{-(b_{SD}+b_{R_kD})/2}}{2^{(-b_{SD}+b_{R_kD})/2} 2\Gamma\left((b_{SD}+b_{R_kD}+2)/2\right)}, \tag{21a}$$

$$P_b^{BDF_k^1}(E) \doteq \int_0^\infty \int_0^{2i_2} f_{I_{SD}}(i_1) f_{I_{R_kD}}(i_2) di_1 di_2. \tag{21b}$$

Therefore, taking into account the asymptotic behavior previously obtained, the expression in Eq. (13) can be simplified as follows

$$P_b^{ADF}(E) \doteq P_b^{BDF_{\min}^0}(E) P_b^{SR_{\min}{}^{0}_{ADF}}(E), \quad b_{R_{\min}D} < \sum_{i=1}^{N_r} b_{SR_i} \tag{22a}$$

$$P_b^{ADF}(E) \doteq \left(1 + \frac{2^{\left(b_{SD}+\sum_{i=1}^{N_r} b_{SR_i}\right)}}{b_{SD}} \sum_{k=1}^{N_r} b_{SR_k} P_b^{BDF_k^1}(E)\right) \cdot P_b^{SD_{ADF}}(E), \quad b_{R_{\min}D} > \sum_{i=1}^{N_r} b_{SR_i} \tag{22b}$$

being $b_{R_{\min}D}$=min$\{b_{R_1D},\ldots,b_{R_{N_r}D}\}$. Taking into account these expressions, the adoption of the ADF cooperative protocol using N_r relays here analyzed translates into a diversity order gain, $G_d(N_r)$, relative to the non-cooperative link *S-D* of

$$G_d(N_r) = 1 + \frac{\min(b_{R_{\min}D}, \sum_{i=1}^{N_r} b_{SR_i})}{b_{SD}}. \tag{23}$$

For the better understanding of the impact of the cooperative FSO communications system here analyzed, the diversity order gain $G_d(N_r)$ in Eq. (23) as a function of the radius of a circumference, d_{RD}, whose center is the node D, is depicted in Fig. 2 for a *S-D* link distance d_{SD}=3 km when different number of relays $N_r = \{1,2,3,4\}$ is considered, as shown in Fig. 1. Furthermore, different scenarios are considered in order to evaluate the performance when different relay-destination link distances are assumed. In this figure, the performance analysis is evaluated for $d_{RD} = d'_{RD}$, $d_{RD} - d'_{RD} = 0.1d_{SD}$ and $d_{RD} - d'_{RD} = 0.2d_{SD}$. Here, the parameters α and β are calculated from Eq. (3a) and Eq. (3b) respectively, with λ=1550 nm. Different weather conditions are here adopted, assuming haze visibility of 4 km with $C_n^2 = 2\times10^{-14}\ m^{-2/3}$ and clear visibility of 16 km with $C_n^2 = 8\times10^{-14}\ m^{-2/3}$, corresponding to moderate and strong atmospheric turbulence conditions, respectively.

Hereinafter, $\varphi^2_{R_kD} \triangleq \varphi^2_{RD}$ and $\varphi^2_{SR_k} \triangleq \varphi^2_{SR}$ are assumed for $k = \{1,2,\ldots,N_r\}$. In Fig. 2(a), the conditions $\varphi^2 > \beta$ is satisfied in both figures for each link and, hence, these results are independent of pointing errors with regards to the diversity order due to the diversity gain only depends of the atmospheric turbulence when this relation is satisfied. Firstly, it can be concluded that the adaptive DF cooperative protocol using relay selection provides a diversity order gain greater than 1 in any case. By other hand, the diversity order gain is a consequence of the inverse relation between the number of relays and the relay-destination link distance. In this way, it is corroborated that the available diversity order is strongly dependent of the relay-destination link distances and the number of relays, achieving a greater diversity gain as the number of relays increases and the relay-destination link distances decrease. Moreover, the diversity order gain presents a maximum when the relation $\beta_{R_{\min}D} = \sum_{i=1}^{N_r} \beta_{SR_i}$ holds. Diversity order gain increases as the relay-destination link distance decreases, being initially dominated by $b_{R_{\min}D}$, i.e., the relay node whose parameter b_{R_kD} is the smallest. Once $b_{R_{\min}D} > \sum_{i=1}^{N_r} b_{SR_i}$ is satisfied, the diversity order gain will be determined by the number of relays. As can be seen in Fig. 2 when $d_{RD} - d'_{RD} = 0.1d_{SD}$ or $d_{RD} - d'_{RD} = 0.2d_{SD}$, only one relay is around of the circumference of radius d_{RD} (outer ring) and the remaining relays are around of the circumference of the radius d'_{RD} (inner ring), being these scenarios representative enough in order to evaluate the diversity order gain when different relay-destination link distances are considered, modelling the worst case. The comparison of these different locations of relays corroborate that changes in diversity performance are negligible, specially over strong atmospheric turbulence or when the link distance is increased, tending to an idealized situation wherein all relays are at the same distance from the destination node. In addition to previous scenario, different values of normalized beam width and normalized jitter of (ω_z/r,σ_s/r)=(5,3) for the *S-D* link are considered in Fig. 2(b). As a result, a greater robustness to the pointing errors for the ADF cooperative protocol is corroborated, increasing the diversity order when pointing errors for the *S-D* link are more severe compared to the *S-R_k* links. The results corresponding to this asymptotic analysis with rectangular pulse shapes and ξ=1 are illustrated in Fig 3, for a source-destination link distance d_{SD}=3 km, together with values of normalized beam width and normalized jitter of (ω_z/r,σ_s/r)=(5,1) and (ω_z/r,σ_s/r)=(10,2). For the sake of simplicity, it is assumed that $d_{RD} = d'_{RD}$ as a consequence of previous conclusions corroborated in Fig. 2. Monte Carlo simulation results are furthermore included as a reference, confirming the accuracy and usefulness of the derived results. Due to the long simulation time involved, simulation results only up to

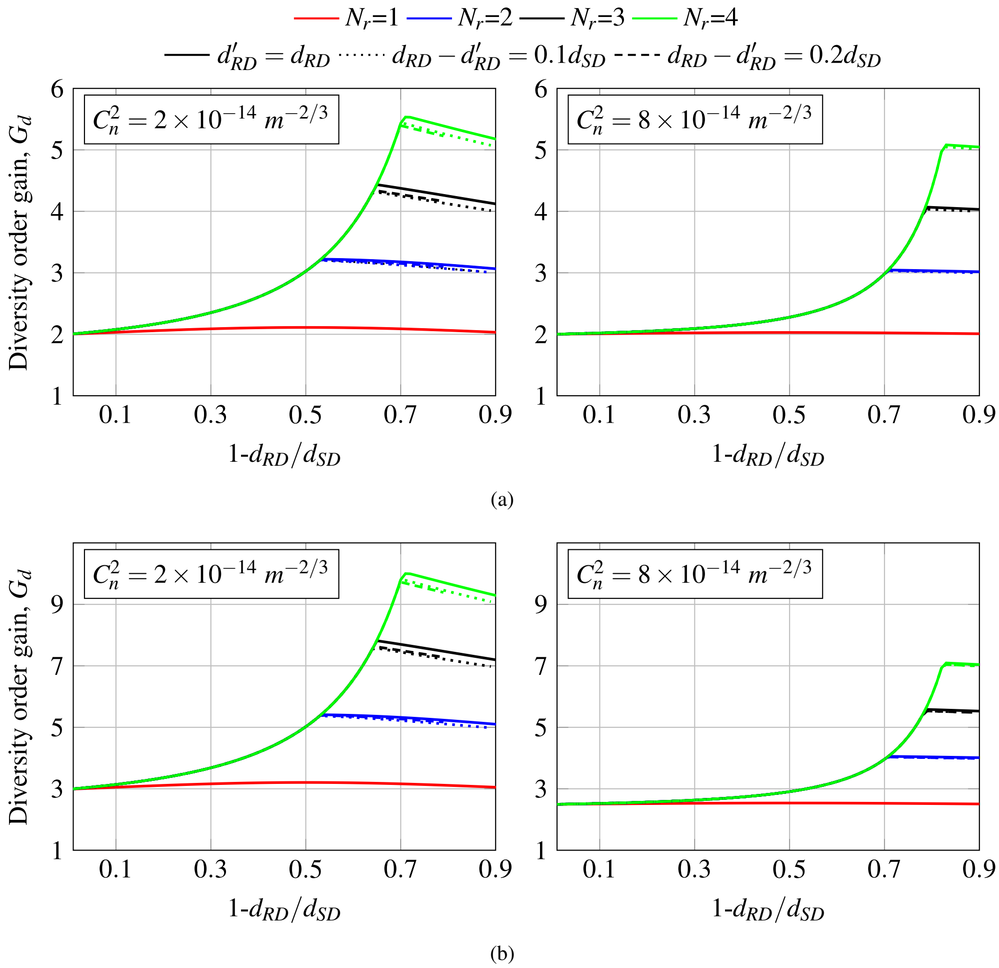
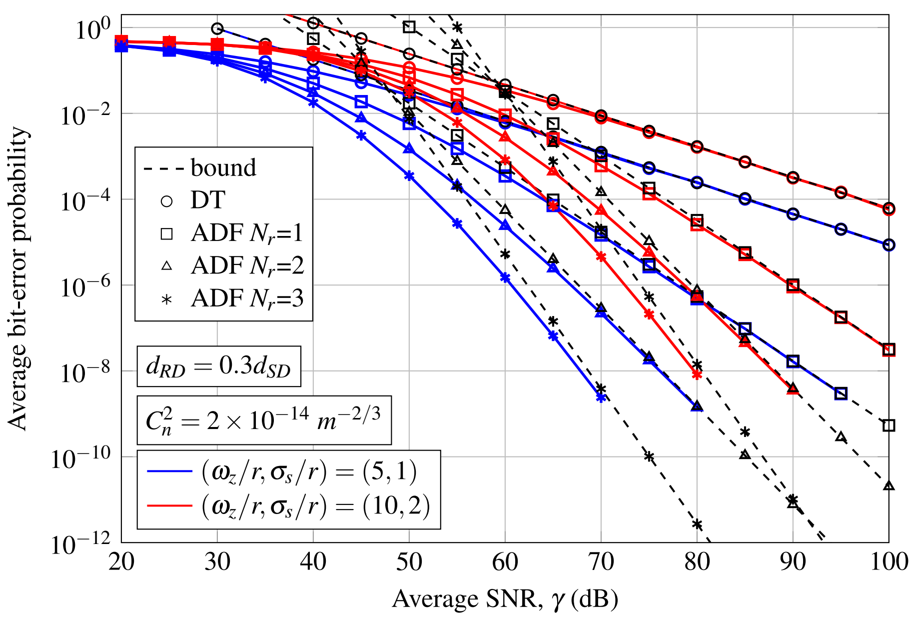
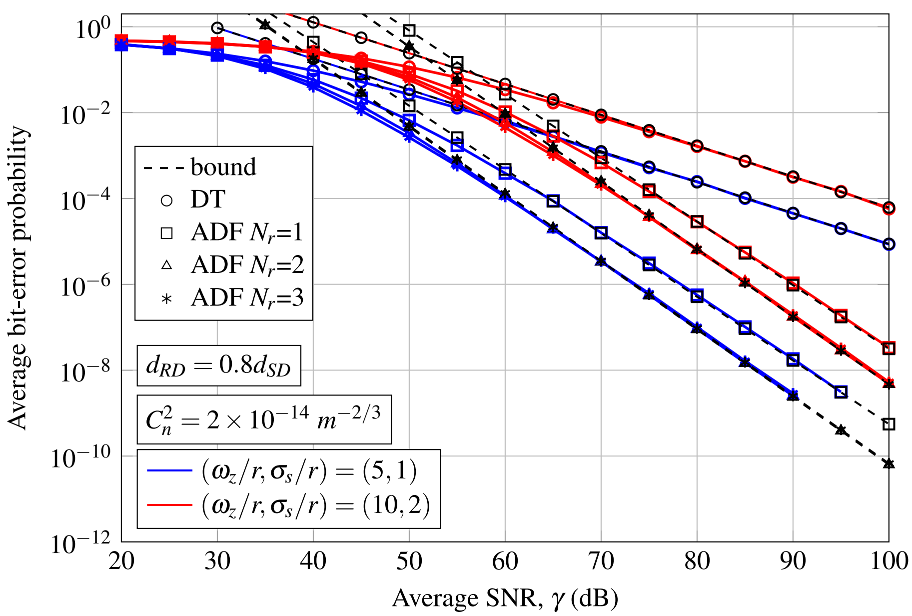


Fig. 2. Diversity order gain G_d for the ADF cooperative protocol for a source-destination link distance of d_{SD}=3 km when different weather conditions and relay locations are assumed. (a) (ω_z/r,σ_s/r)=(5,1) for each link and (b) (ω_z/r,σ_s/r)=(5,1) for the *S*-R_k and R_k-*D* links, and (ω_z/r,σ_s/r)=(5,3) for the *S*-*D* link.

BER=10^{-9} are included. Additionally, we also consider the performance analysis for the direct transmission (non-cooperative link *S*-*D*) to establish the baseline performance. It can be corroborated that these BER results are in excellent agreement with previous results shown in Fig. 2 in relation to the diversity order gain achieved for the ADF cooperative protocol when different number of relays $N_r = \{1,2,3\}$ are assumed. In this sense, we can see diversity gains of 2.06, 2.18 and 2.18 for a *R*-*D* link distance $d_{RD} = 0.8d_{SD}$, or diversity gains of 2.08, 3.17 and 4.37 for a *R*-*D* link distance $d_{RD} = 0.3d_{SD}$, when the number of relays is 1, 2 and 3, respectively. Both cases are considered in Eq. (22), being Eq. (22a) and Eq. (22b) the bounds corresponding to each FSO scenario, respectively. Next, G_d is depicted in Fig. 4 for a source-destination link distance of d_{SD}=3 km and $d_{RD} = d'_{RD}$, when normalized beam width of ω_z/r=5 and different values of normalized jitter σ_s/r=$\{1,2,3\}$ are assumed, in order to contrast the impact of pointing errors when the condition $\varphi^2 > \beta$ is not satisfied for each link. It can be



(a)



(b)

Fig. 3. BER performance when different number of relays N_r=$\{1,2,3\}$ and source-destination link distance of d_{SD}=3 km are assumed. Different relay locations of (a) d_{RD}=0.3d_{SD} and (b) d_{RD}=0.8d_{SD} are assumed together with values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = \{(5,1),(10,2)\}$.

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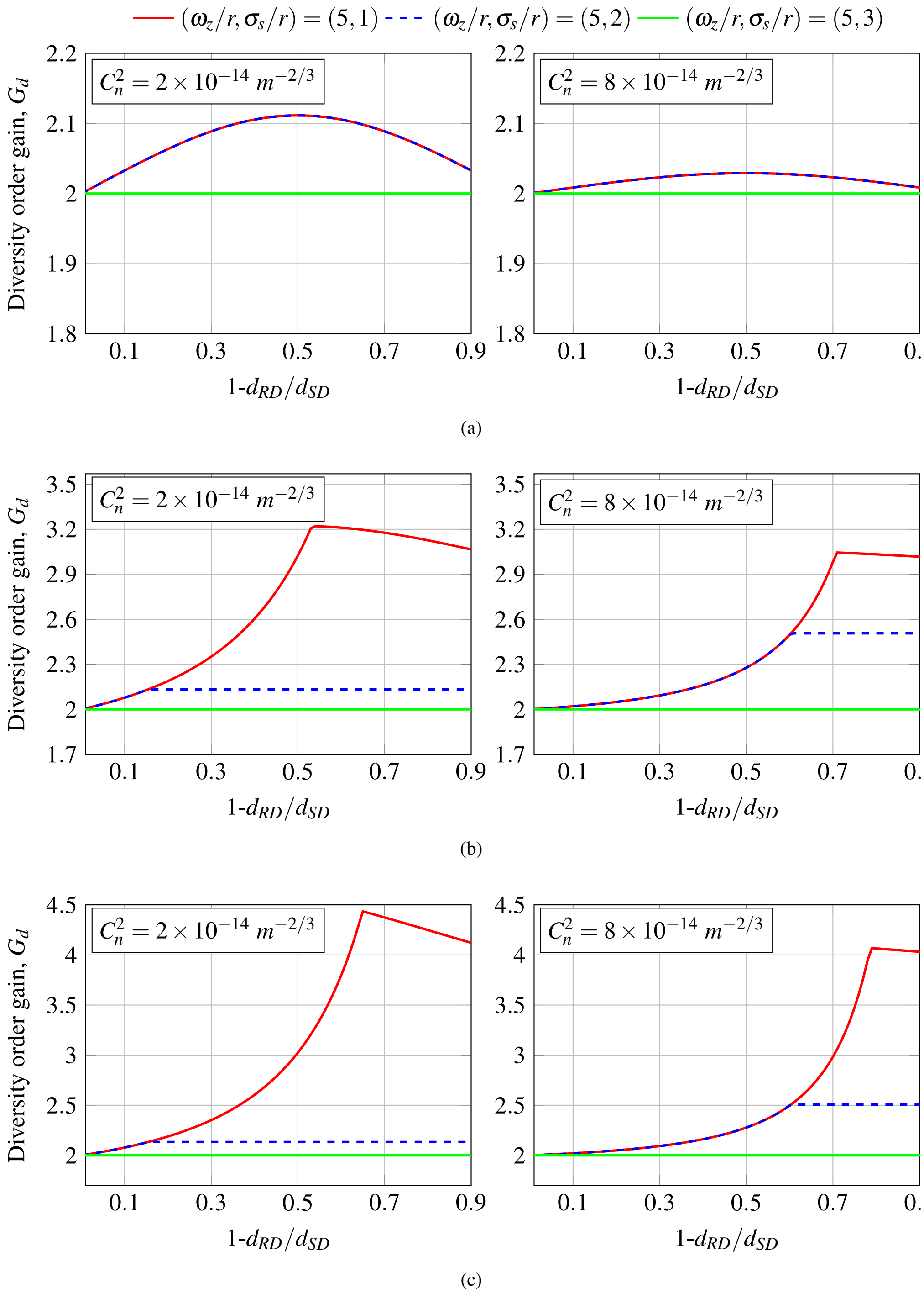

Fig. 4. Diversity order gain G_d for the Adaptive-DF cooperative protocol for a source-destination link distance of d_{SD}=3 km when (a) 1, (b) 2 and (c) 3 relays are assumed together with different values of normalized beam width of ω_z/r= 5 and normalized jitter of σ_s/r=$\{1,2,3\}$.

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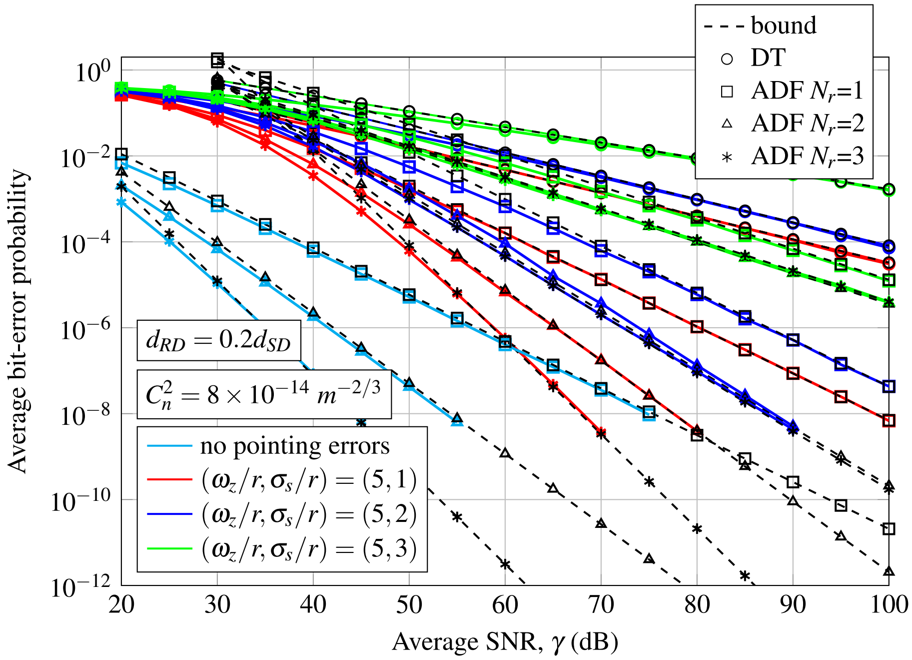

Fig. 5. BER performance when different number of relays N_r=$\{1,2,3\}$ and source-destination link distance of d_{SD}=3 km are assumed when d_{RD}=0.2d_{SD} together with a normalized beam width of ω_z/r= 5 and normalized jitter of σ_s/r=$\{1,2,3\}$ as well as the FSO scenario without pointing errors.

observed that diversity gains even greater than 2, 3 and 4 for N_r set to 1, 2 and 3, respectively, are achieved when $(\omega_z/r, \sigma_s/r) = (5,1)$, not being affected by pointing errors regardless of the weather conditions. Moreover, in Fig. 4(b) and Fig. 4(c), for values of normalized beam width and normalized jitter of (ω_z/r,σ_s/r)=(5,2), the condition $\varphi^2 > \beta$ is not satisfied for the relay-destination link when the relation $\varphi_{RD}^2 = \beta_{R_kD}$ holds. In this case, the diversity gain is limited mainly by the pointing errors of the relay-destination links. Under strong atmospheric turbulence conditions the ADF relaying scheme presents a greater robustness to the pointing errors, achieving the maximum diversity gain when $d_{RD} \approx 0.4d_{SD}$ instead of $d_{RD} \approx 0.85d_{SD}$ assuming moderate atmospheric turbulence. Finally, the FSO scenario in which the condition $\varphi^2 > \beta$ is not satisfied for each link is depicted in Fig. 4(c) for values of normalized beam width and jitter of (ω_z/r,σ_s/r)=(5,3) and, hence, the diversity gain is saturated to the value of 2, as can be deduced from Eq. (23) as follows

$$G_d(N_r) = 1 + \frac{\min\left(\varphi_{RD}^2, N_r\varphi_{SR}^2\right)}{\varphi_{SD}^2} = 1 + \frac{\min\left(\varphi^2, N_r\varphi^2\right)}{\varphi^2} = 2, \tag{24}$$

being an independent value of the number of relays or the relay locations. These conclusions are contrasted in Fig. 5, wherein BER performance is displayed for a source-destination link distance of d_{SD}=3 km when *R-D* link distance $d_{RD} = 0.2d_{SD}$ and the number of relays is N_r=$\{1,2,3\}$, together with values of normalized beam width of ω_z/r= 5 and normalized jitter of σ_s/r=$\{1,2,3\}$. As before, the performance analysis for the direct transmission is considered to establish the baseline performance. These BER results are in excellent agreement with previous results shown in Fig. 4 in relation to the diversity order gain achieved for this cooperative

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FSO communication system with multiple relays where pointing errors are presented. These results show that the impact of pointing errors is less severe for the ADF cooperative protocol compared with other similar cooperative schemes or equivalent multiple-input single-output (MISO) systems [22]. In this sense, it can be seen that diversity gains of 2.01, 2.01 and 2 for a number of relays of $N_r = 1$, 3.03, 2.5 and 2 for a number of relays of $N_r = 2$, and 4.06, 2.5 and 2 for a number of relays of $N_r = 3$ are achieved with values of normalized jitter of $\sigma_s/r = \{1,2,3\}$, respectively. As also displayed in Fig. 3, simulation results corroborate that asymptotic expressions here obtained lead to simple bounds on the bit error probability that get tighter over a wider range of SNR as the turbulence strength increases. Finally, we conclude the paper by evaluating that the impact of the pointing errors effects translates into a coding gain disadvantage. Knowing that the average BER behaves asymptotically as $(G_c\gamma\xi)^{-G_d}$, where G_d and G_c denote diversity order and coding gain, respectively. Once the condition $\varphi^2 > \beta$ is satisfied for each link, it can be convenient to compare with the BER performance obtained in a similar context without pointing errors. Taking into account that the greatest diversity order gain is achieved when the relation $\beta_{R_{\min}D} > \sum_{i=1}^{N_r} \beta_{SR_i}$ holds, and knowing that the impact of pointing errors in our analysis can be suppressed by assuming $A_0 \to 1$ and $\varphi^2 \to \infty$ [17], the corresponding asymptotic expression can be easily derived from Eq. (22b) as follows

$$P_b^{ADF}(E) \doteq \left(1 + \frac{2^{\left(\beta_{SD}+\sum_{i=1}^{N_r}\beta_{SR_i}\right)}}{\beta_{SD}} \sum_{k=1}^{N_r} \beta_{SR_k} P_b^{BDF_k^1}(E)\right) \cdot P_b^{SD_{ADF}^{npe}}(E), \tag{25}$$

As can be numerically corroborated, the effects of the pointing errors on $P_b^{BDF_k^1}(E)$ can be considered negligible in this FSO scenario and, hence, the first factor in Eq. (22b) and Eq. (25) can be suppressed for the computation of the coding gain disadvantage. $P_b^{SD_{ADF}^{npe}}(E)$ is obtained from Eq. (17) when misalignment fading is not present, wherein the corresponding parameter a_m is given by

$$a_m^{npe} = \frac{(\alpha_m\beta_m)^{\beta_m}\Gamma(\alpha_m-\beta_m)}{L_m^{\beta_m}\Gamma(\alpha_m)\Gamma(\beta_m)}. \tag{26}$$

In Fig. 5, BER performance in the same FSO context without pointing errors is also displayed. From this asymptotic analysis and taking into account the coding gain in Eq. (22b), the impact of the pointing errors translates into a coding gain disadvantage, $D_{pe}[dB]$, as follows

$$D_{pe}[dB] \approx \frac{20}{\beta_{SD}+\sum_{i=1}^{N_r}\beta_{SR_i}} \log_{10}\left(\frac{\varphi_{SD}^2}{A_0^{\beta_{SD}}(\varphi_{SD}^2-\beta_{SD})} \prod_{i=1}^{N_r} \frac{\varphi_{SR_i}^2}{A_0^{\beta_{SR_i}}(\varphi_{SR_i}^2-\beta_{SR_i})}\right). \tag{27}$$

According to this expression, it can be observed in Fig. 5 that a coding gain disadvantage of 23.75 decibels is achieved for a value of $(\omega_z/r, \sigma_s/r) = (5,1)$ regardless of the number of relays. This result can be justified from the fact that $\beta_{SD} \approx \beta_{SR_1} \approx \cdots \approx \beta_{SR_{N_r}}$ holds in this scenario and, hence, Eq. (27) can be simplified as follows

$$D_{pe}[dB] \approx \frac{20}{(N_r+1)\beta_{SD}} \log_{10}\left(\frac{A_0^{-\beta_{SD}}\varphi_{SD}^2}{(\varphi_{SD}^2-\beta_{SD})}\right)^{N_r+1} = \frac{20}{\beta_{SD}} \log_{10}\left(\frac{A_0^{-\beta_{SD}}\varphi_{SD}^2}{(\varphi_{SD}^2-\beta_{SD})}\right), \tag{28}$$

obtaining the coding gain disadvantage corresponding to the direct transmission based on the non-cooperative source-destination link.

4. Conclusions

In this paper, a novel adaptive cooperative protocol with multiple relays using detect-and-forward over atmospheric turbulence channels with pointing errors is proposed. The adaptive DF cooperative protocol here analyzed is based on the selection of the optical path, source-destination or different source-relay links, with a greater value of fading gain or irradiance, maintaining a high diversity order. Closed-form asymptotic expressions are obtained for this cooperative protocol using relay selection when the irradiance of the transmitted optical beam is susceptible to either a wide range of turbulence conditions (moderate to strong), following a gamma-gamma distribution of parameters α and β, or pointing errors, following a misalignment fading model, where the effect of beam width, detector size and jitter variance is considered. The superiority of this cooperative protocol is related to the significant improvement in diversity gain, being this dependent of the relay-destination link distance and number of relays. Regarding the more severe impact of pointing errors, it is concluded that the diversity gain is saturated to 2 when the parameter $\varphi^2 < 1$ since the condition $\varphi^2 > \beta$ is not satisfied for any link, regardless of the relay location and number of relays. As a result, the ADF relaying scheme here analyzed presents a greater order diversity gain if compared with similar cooperative schemes or equivalent MIMO systems. Finally, simulation results are further demonstrated to confirm the accuracy and usefulness of the derived results, showing that asymptotic expressions.

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