Estimating radial railway network improvement with a CAS

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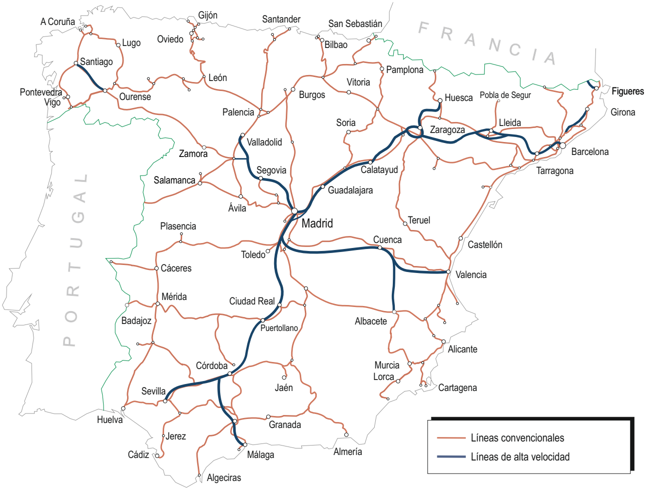
Two different gauges

- The Spanish railway network is very complex, with two different track gauges:
  - The bread classic **Iberian** track gauge (1667mm) and
  - The **international gauge** (1435mm) used in the extensive high speed network.
- There is also a small narrow gauge network.
- There are gauge changeovers at several points that connect both subnetworks.

Two gauges map



Characteristics 1

- **Adif** company takes care of the infrastructure.
- **Renfe** company operates most trains.
- Only China has nowadays a longer high speed railway network.
- Moreover, the longest high speed railway service in Europe is operated in Spain: the 1121km Barcelona–Málaga, operated at an average speed of 204km/h.
- The high speed railway network has grown very quickly, as the first line (Madrid–Seville) was opened only in 1992.

Characteristics 2

- All new lines have been built with double track and top technologies ($\geq$ 300km/h track design, *LZB* or *ERTMS* traffic management system, 25000KV AC electrification, etc.).
- *Renfe*'s rolling stock is very flexible, with some dual gauge trains and multiple units (using two different gauge change systems: *Talgo* and CAF).
- Many locomotives and multiple units can read different signalling systems (*ASFA*, *ASFA 200*, *LZB*, *EBICAB*, *ERTMS*), are multi-voltage and even hybrid rolling stock has been developed.

How to grow

- There are controversial opinions among experts regarding how the network should grow, especially after the cuts due to the economic crisis (let us underline that the completion of the NW line, the N line, the Spanish side of Madrid–Lisbon line and the extensions of the SE line, are under work now).
- An alternative could be to build very high speed trunks followed by *not so high speed* (for example 200km/h) antennas. Although designed for high speed traffic, these antennas could be (initially) single track if the expected traffic was low. Moreover, the circulation of high speed trains in these antennas would not exclude the circulation of freight trains, regional trains or even commuter trains.

Two research lines

Due to the controversy mentioned, we performed some research in order to easily compare the different alternatives for routing trains and for building new infrastructures in the Spanish railway network. We followed two research lines:

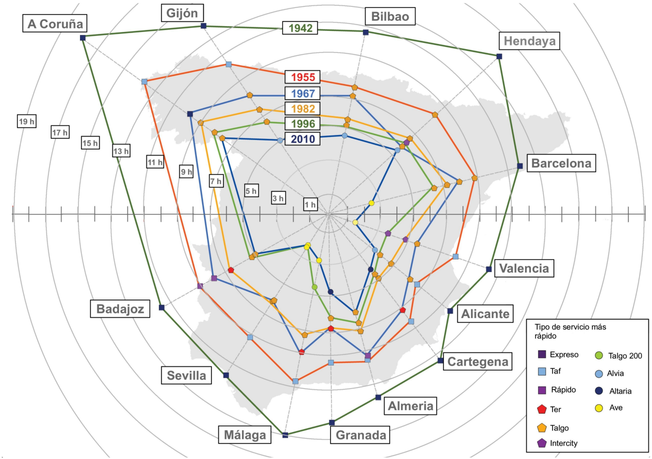
1. We developed a computer package that is able to calculate precise timings, consumptions, costs, emissions, best routes, etc., for each piece of *Renfe*'s rolling stock running on *Adif*'s lines.
2. We developed what we have called **isochrone circle graphs** and a **geometric index** for radial railway networks improvement estimation, that can be very useful for decision taking regarding the improvement of railway lines.

Isochrone circle graphs 1

Inspired by:

- **Pie charts** (also known as *circle graphs*), where the radii of all sectors is equal.
- **Polar area diagrams**. These diagrams are similar to usual pie charts, but sectors are equal angles and their area is adjusted changing their radii (instead of their amplitude).
- **Anamorphosis maps** (also known as *central point cartograms* or *distance cartograms*). In these maps or cartograms the geometry is distorted according to the time that it takes to travel to different peripheral destinations from a central origin.

Isochrone circle graphs 2 (Anamorphosis map from Madrid)



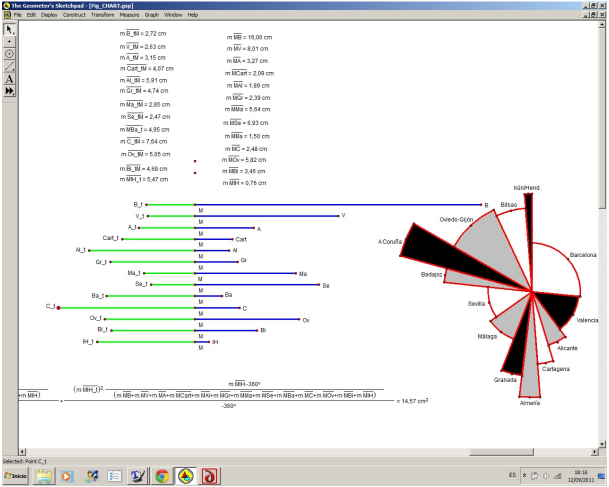
Isochrone circle graphs 3

- Our **isochrone circle graphs** take ideas from these three graphic representations (both amplitudes and radii are different for each sector and the shape obtained is relatively related to an anamorphosis map but substituting land area by population).
- The graph is formed by contiguous sectors. The radius of each sector is proportional to the timing to the corresponding destination (not to its square root). Meanwhile, the amplitudes of the sectors are proportional to the population served (so they are not equal).
- Consequently, the area of an *isochrone circle graph* gives an idea of the average speed in the network (considering the population affected).

Isochrone circle graphs 4

- Finally, 1 minus the quotient of the area of the *isochrone circle graph* if a proposed timetable change was implemented and the area of the original *isochrone circle graph* (expressed as a percentage) is what we denoted the *geometric index* of the proposed radial railway network improvement.
- These ideas were detailed in a previous paper, that was illustrated with a *sketch* constructed with the Dynamic Geometry System (DGS) *The Geometer's Sketchpad (GSP)*, that used sliders to change the input parameters (timing to each peripheral destination and population of these destinations) and was able to build the corresponding *isochrone circle graph*.

Isochrone circle graphs 5 (with The Geometer's Sketchpad)



Isochrone circle graphs 6

- Although very comfortable to use, altering the number of peripheral destinations considered required to construct a complete new sketch (!), what is very laborious and requires of some knowledge of *GSP*.
- In another related work regarding the shape of the Spanish high speed railway network, an algorithmic approach is developed and applied to the Spanish case, resulting in a slightly different best solution for the high speed railway network than the one carried out by the Spanish governments.

The next step 1

- We could find no geometrical solution to the problem with the fixed number of destinations in the GSP approach stated previously. Therefore we considered the possibility to begun from scratch and design and implement a complete new package in a computer algebra system (CAS) like *Maple*.
- It should take as input the lists of destinations, timings and populations and should build the corresponding **isochrone circle graphs** and perform all the corresponding calculations (so that not only the timings and populations were free but also the number of peripheral destinations).

The next step 2

- Moreover, such new approach should allow the user to perform symbolic computations with the output data (i.e., it should allow to introduce input parameters to the formulae, that were carried along the subsequent computations). Therefore, inverse problems, such as obtaining the timing improvements required for fulfilling a certain goal, could now be addressed.
- Another add on would be the possibility to plot functions depending on these parameters.

Design of the CAS approach

Modern CAS are usually used from worksheets. We consider that using global variables for the input data can be more convenient than introducing long inputs to the main procedure.

The global data variables considered (lists) are:

- Boolean variable reflecting if there will be symbolic values in the input data: `symbolic` (YES / NO),
- names of destinations: list `NID`,
- timings to destinations: list `TID`,
- population of destinations: list `PID`

(the ordering in all lists is: North 1st, clockwise).

Implementation of the CAS approach

- We have chosen **Maple** because of our expertise with this CAS, but any of the big CAS could be used instead. *Maple 16* includes command `sector` (`pieslice` in *Maple* previous versions) specialized in plotting sectors, that although not strictly necessary, saves programming time.
- The algorithm and code will be described and detailed in the paper to appear in the proceedings.

Examples of use

- Numeric examples.
- Symbolic examples.

Numeric Examples

ERL, 7-II-2013, 16-IV-2013, 17-V-2013

Initialize:

```
> restart;
> with(plottools):
> with(plots):
> #Digits:=5:
```

Load the code:

```
> read(`C:/CONGRES/2013/FEMTEC2013-MapaIsocronasCAS/Maple/Isochrone.mpl`);
```

Use of symbolic data as input (if YES then no plot is generated):

```
> symbolic:=NO:      #YES or NO
```

Numeric Examples: Example 1

Example 1: Renfe 2011 (RACSAM article)

Input data:

Names of destinations Input Data (North 1st, clockwise)

```
> NID:=[ Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
```

Timings to destinations Input Data (North 1st, clockwise) Timings given in the form: [hours, minutes]

```
> TID:=[ [5, 25], [2, 43], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [4, 57], [7, 38], [5,  3], [4, 41] ]:
```

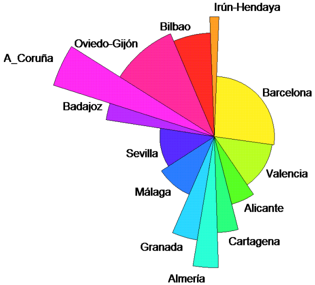
Population of destinations Input data (North 1st, clockwise):

```
> PID:=[ 74938, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
```

Computations of the example:

```
> pie();
> display(ISOPLOT);
```

Numeric Examples




```
> ARSEC;
```

[[*Irún – Hendaya*, 11.19404613], [*Barcelona*, 60.84581605], [*Valencia*, 28.57092345], [*Alicante*, 16.89396250], [*Cartagena*, 20.47450758], [*Almería*, 33.86556247], [*Granada*, 27.27926103], [*Málaga*, 23.50965790], [*Sevilla*, 21.81405935], [*Badajoz*, 18.75900069], [*A_Coruña*, 72.99066199], [*Oviedo – Gijón*, 76.15742834], [*Bilbao*, 39.43988337]]

```
> totalArea();
```

451.7947708

Numeric Examples

Example 2 (of Improvement Index): (example at the end of RACSAM article: time to A Coruña improved to 4 h)

We store the original area just computed as "totalArea1":

```
> totalArea1:=totalArea();
```

$totalArea1 := 451.7947708$

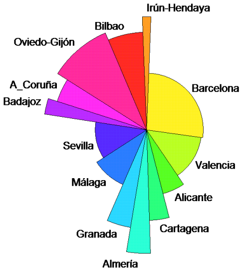
```
> TID:=[ [5, 25], [2, 43], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [4, 57], [4, 30], [5,  3], [4, 41] ]:
> pie();
> totalArea2:=totalArea();
```

$totalArea2 := 404.1708031$

```
> 100*(1-totalArea2/totalArea1);
```

10.54106218

```
> display(ISOPLOT);
```



Numeric Examples

Example 3 (other example of Improvement Index): (example at the end of RACSAM article: time to Badajoz improved to 3 h)(peq. dif. GSP)

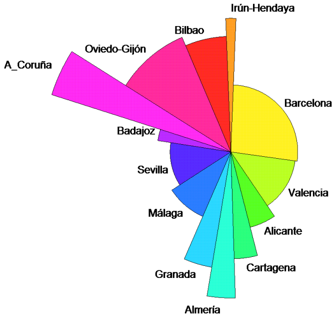
```
> TID:=[ [5, 25], [2, 43], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [3, 00], [7, 38], [5,  3], [4, 41] ]:
> pie();
> totalArea3:=totalArea();
```

$$totalArea3 := 439.9261285$$

```
> 100*(1-totalArea3/totalArea1);
```

$$2.62699860$$

```
> display(ISOPLOT);
```



Numeric Examples

<u>Example 4 (Changing the number of destinations):</u> let us specify the Girona passengers.

```
> NID:=[ Irún-Hendaya, Gerona, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
> TID:=[ [5, 50], [3, 32], [2, 30], [1, 35], [3, 11], [4, 54], [6, 22],
>        [4, 25], [2, 20], [2, 20], [5, 27], [6, 14], [5, 20], [4, 47] ]:
> PID:=[ 74938, 97198, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
```

Computations of the example:

```
> pie();
> ARSEC;
```

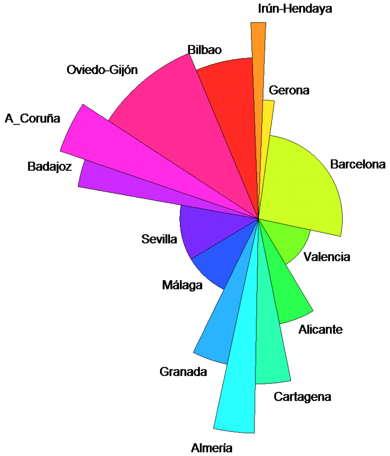
[[*Irún – Hendaya*, 12.77850123], [*Gerona*, 6.080940900], [*Barcelona*, 50.71792647], [*Valencia*, 10.16671791], [*Alicante*, 16.98236269], [*Cartagena*, 25.76824794], [*Almería*, 38.59684206], [*Granada*, 23.37820321], [*Málaga*, 15.51077935], [*Sevilla*, 19.21288533], [*Badajoz*, 22.38286967], [*A_Coruña*, 47.90744512], [*Oviedo – Gijón*, 83.60849960], [*Bilbao*, 40.49582356]]

```
> totalArea();
```

413.5880451

```
> display(ISOPLOT);
```

Numeric Examples



Numeric Examples

Example 5 (Changing the number of destinations): imagine that Bilbao and Irún-Hendaya destinations should be merged in the study and that the average of timigs could be asigned to the new "destination". (not used)

```
> NID:=[ Bilbao-Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón ]:
> TID:=[  [(5+4)/2, (25+41)/2], [2, 43], [2, 38], [3,  9], [4, 20], [5, 55],
>         [4, 44], [2, 51], [2, 28], [4, 57], [7, 38], [5,  3] ]:
> PID:=[ (74938+353187)/2, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555 ]:
```

Computations of the example:

```
> pie();
> ARSEC;
```

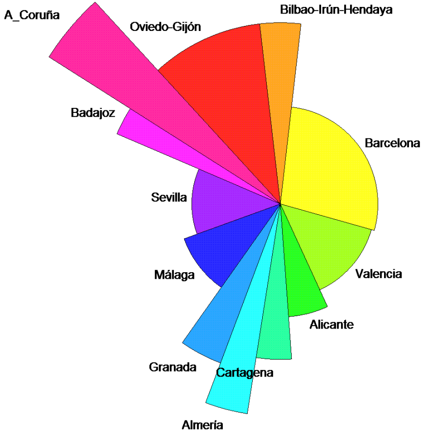
[[*Bilbao – Irún – Hendaya*, 28.80605772], [*Barcelona*, 63.06238979], [*Valencia*, 29.61174370], [*Alicante*, 17.50939862], [*Cartagena*, 21.22038063], [*Almería*, 35.09926300], [*Granada*, 28.27302686], [*Málaga*, 24.36609952], [*Sevilla*, 22.60873142], [*Badajoz*, 19.44237894], [*A_Coruña*, 75.64966453], [*Oviedo – Gijón*, 78.93179417]]

```
> totalArea();
```

444.5809288

```
> display(ISOPLOT);
```

Numeric Examples



FIN

>

Symbolic Examples

ERL, 12-II-2013, 17-V-2013, 18-V-2013 **(Maple 16)**

Initialize:

```
> restart;
> with(plottools):
> with(plots):
```

Remove the # afterwards in case you would like to round the floating point numbers with that number of digits:

```
> #Digits:=5:
```

Load the code:

```
> read(`C:/CONGRES/2013/FEMTEC2013-MapaIsocronasCAS/Maple/Isochrone.mpl`);
```

Symbolic:

In th examples below we shall introduce parameters in the computations. We have to inform the system with a:

```
> symbolic:=YES:
```

Symbolic Examples

Symbolic Example 1: m is in this example the improvement in the Barcelona relation (in minutes)

```
> symbolic:=YES:     #YES or NO
> NID:=[ Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
> TID:=[ [5, 25], [2, 43-m], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [4, 57], [7, 38], [5,  3], [4, 41] ]:
> PID:=[ 74938, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
> pie();
```

Evolution of the influence of the improvement as a function of m:

```
> ARSEC;
```

$[[\mathit{Irún}-\mathit{Hendaya}, 11.19404613], [\mathit{Barcelona}, 8.244380210\,(2.716666667-0.01666666667\,m)^2], [\mathit{Valencia}, 28.57092345],$
$[\mathit{Alicante}, 16.89396250], [\mathit{Cartagena}, 20.47450758], [\mathit{Almería}, 33.86556247], [\mathit{Granada}, 27.27926103], [\mathit{Málaga}, 23.50965790],$
$[\mathit{Sevilla}, 21.81405935], [\mathit{Badajoz}, 18.75900069], [\mathit{A\_Coruña}, 72.99066199], [\mathit{Oviedo}-\mathit{Gijón}, 76.15742834], [\mathit{Bilbao}, 39.43988337]]$

```
> totalArea();
```

$$451.7947709-0.7465744304\,m+0.002290105615\,m^2$$

let us format it as a Maple function (and plot it):

```
> ImprBarna:=x->subs(m=x,totalArea());
```

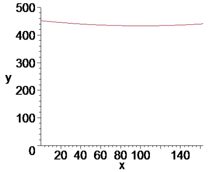
$$\mathit{ImprBarna} := x \rightarrow \mathrm{subs}(m=x, \mathrm{totalArea}(\ ))$$

```
> ImprBarna(x);
```

$$451.7947709-0.7465744304\,x+0.002290105615\,x^2$$

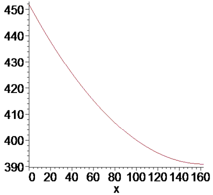
Symbolic Examples

```
> plot(ImprBarna(x),x=0..163,y=0..500,scaling=unconstrained);
```



Let us show only the range of the y axis where the function has values (Maple does it by default):

```
> plot(ImprBarna(x),x=0..163,scaling=unconstrained);
```



(the improvement has a trivial upper bound, as expected: when the time is reduced to 0).

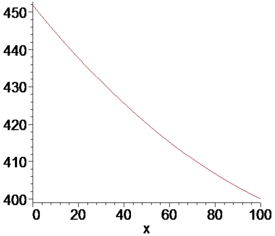
Symbolic Examples

When is the improvement maximum?

```
> diff(ImprBarna(x),x);
```

$$-0.7465744304 + 0.004580211230\, x$$

```
> plot(ImprBarna(x),x=0..100);
```



(the same improvemt has more effect when the times are bad than when the times are good).

Symbolic Examples

Symbolic Example 2: [t1, t2] is in this example the timing in the Barcelona relation in [hours,minutes] (instead of introducing an improvement of m minutes, the new timing is introduced)

```
> symbolic:=YES:     #YES or NO
> NID:=[ Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
> TID:=[ [5, 25], [t1, t2], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [4, 57], [7, 38], [5,  3], [4, 41] ]:
> PID:=[ 74938, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
> pie();
```

Evolution of the influence of the improvement as a function of t1 and t2:

```
> ARSEC;
```

[[*Irún – Hendaya*, 11.19404613], [*Barcelona*, 8.244380210 $(t1 + 0.01666666667\, t2)^2$], [*Valencia*, 28.57092345], [*Alicante*, 16.89396250], [*Cartagena*, 20.47450758], [*Almería*, 33.86556247], [*Granada*, 27.27926103], [*Málaga*, 23.50965790], [*Sevilla*, 21.81405935], [*Badajoz*, 18.75900069], [*A_Coruña*, 72.99066199], [*Oviedo – Gijón*, 76.15742834], [*Bilbao*, 39.43988337]]

```
> totalArea();
```

$$390.9489548 + 8.244380210\, t1^2 + 0.2748126737\, t1\, t2 + 0.002290105615\, t2^2$$

Now the function is a function of two variables:

```
> ImprBarna:=(x1,x2)->subs(t1=x1,t2=x2,totalArea());
```

$$ImprBarna := (x1, x2) \rightarrow \mathrm{subs}(t1 = x1, t2 = x2, \mathrm{totalArea}(\ ))$$

```
> ImprBarna(x1,x2);
```

$$390.9489548 + 8.244380210\, x1^2 + 0.2748126737\, x1\, x2 + 0.002290105615\, x2^2$$

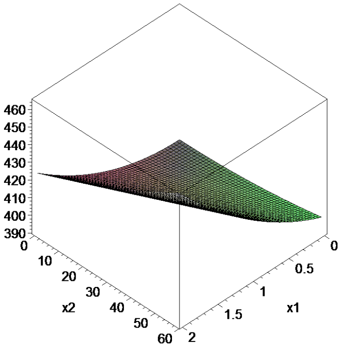
For example, if the timing to Barcelona was improved to 2 h 10 m...

```
> ImprBarna(2,10);
```

$$429.6517397$$

Symbolic Examples

```
> plot3d(ImprBarna(x1,x2),x1=0..2,x2=0..59,axes=boxed,scaling=unconstrained);
```



Symbolic Examples

Symbolic Example 3: m is in this example the improvement percentage in the Barcelona relation

```
> symbolic:=YES:     #YES or NO
> NID:=[ Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
> TID:=[ [5, 25], [(1 - m/100)*2, (1 - m/100)*43], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [4, 57], [7, 38], [5,  3], [4, 41] ]:
> PID:=[ 74938, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
> pie();
```

Evolution of the influence of the improvement as a function of m (m a percentage):

```
> ARSEC;
```

[[*Irún – Hendaya*, 11.19404613], [*Barcelona*, 8.244380210 $(2.716666667 - 0.02716666667\, m)^2$], [*Valencia*, 28.57092345], [*Alicante*, 16.89396250], [*Cartagena*, 20.47450758], [*Almería*, 33.86556247], [*Granada*, 27.27926103], [*Málaga*, 23.50965790], [*Sevilla*, 21.81405935], [*Badajoz*, 18.75900069], [*A_Coruña*, 72.99066199], [*Oviedo – Gijón*, 76.15742834], [*Bilbao*, 39.43988337]]

```
> totalArea();
```

$$451.7947709 - 1.216916321\, m + 0.006084581607\, m^2$$

```
> ImprBarna:=x->subs(m=x,totalArea());
```

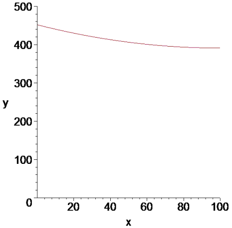
$$ImprBarna := x \to \mathrm{subs}(m = x, \mathrm{totalArea}(\ ))$$

```
> ImprBarna(x);
```

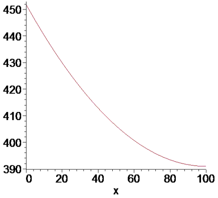
$$451.7947709 - 1.216916321\, x + 0.006084581607\, x^2$$

Symbolic Examples

```
> plot(ImprBarna(x),x=0..100,y=0..500,scaling=unconstrained);
```




```
> plot(ImprBarna(x),x=0..100,scaling=unconstrained);
```



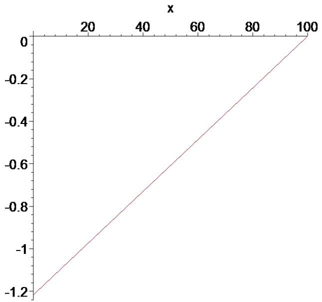
Symbolic Examples

When is the improvement maximum?

```
> diff(ImprBarna(x),x);
```

$$-1.216916321 + 0.01216916321\, x$$

```
> plot(%,x=0..100);
```



(as shown above, when timings are bad).

Symbolic Examples

Symbolic Example 4: Inverse problem (Barcelona)

```
> ImprBarna(0);
```

451.7947709

How much should the timings to Barcelona be improved in order to improve the whole timings in Spain a 5% ?

```
> solve( ImprBarna(x)=(95/100)*ImprBarna(0) , x);
```

179.2930018, 20.70699816

they should be improved a 20.70% (the other solution : 179.29% makes no sense).

How much should the timings to Barcelona be improved in order to improve the whole timings in Spain a 20% ?

```
> solve( ImprBarna(x)=(80/100)*ImprBarna(0) , x);
```

$99.99999997 + 69.64538370\, I,\ 99.99999997 - 69.64538370\, I$

there is no real solution (it is not possible to achieve this global improvement acting only on the Barcelona relation).

Symbolic Examples

Symbolic Example 5: m is in this example the improvement percentage in the Badajoz relation

```
> symbolic:=YES:     #YES or NO
> NID:=[ Irún-Hendaya, Barcelona, Valencia, Alicante, Cartagena, Almería,
>        Granada, Málaga, Sevilla, Badajoz, A_Coruña, Oviedo-Gijón, Bilbao ]:
> TID:=[ [5, 25], [2, 43], [2, 38], [3,  9], [4, 20], [5, 55],
>        [4, 44], [2, 51], [2, 28], [(1 - m/100)*4, (1 - m/100)*57], [7, 38], [5,  3], [4, 41] ]:
> PID:=[ 74938, 1619337, 809267, 334418, 214165, 190013,
>       239154,  568507, 704198, 150376, 246047, 586555, 353187 ]:
> pie();
```

Evolution of the influence of the improvement as a function of m (m a percentage):

```
> ARSEC;
```

[[*Irún – Hendaya*, 11.19404613], [*Barcelona*, 60.84581605], [*Valencia*, 28.57092345], [*Alicante*, 16.89396250], [*Cartagena*, 20.47450758], [*Almería*, 33.86556247], [*Granada*, 27.27926103], [*Málaga*, 23.50965790], [*Sevilla*, 21.81405935], [*Badajoz*, $0.7655953754\,(4.950000000 - 0.04950000000\,m)^2$], [*A_Coruña*, 72.99066199], [*Oviedo – Gijón*, 76.15742834], [*Bilbao*, 39.43988337]]

```
> totalArea();
```

$$451.7947708 - 0.3751800137\,m + 0.001875900069\,m^2$$

```
> ImprBad:=x->subs(m=x,totalArea());
```

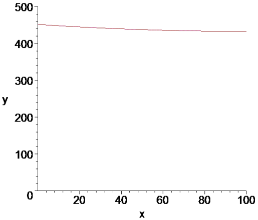
$$ImprBad := x \to \mathrm{subs}(m = x, \mathrm{totalArea}(\ ))$$

```
> ImprBad(x);
```

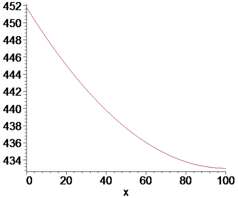
$$451.7947708 - 0.3751800137\,x + 0.001875900069\,x^2$$

Symbolic Examples

```
> plot(ImprBad(x),x=0..100,y=0..500,scaling=unconstrained);
```




```
> plot(ImprBad(x),x=0..100,scaling=unconstrained);
```



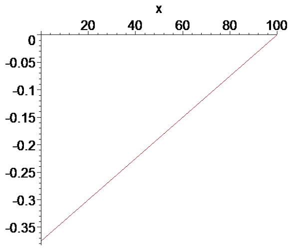
Symbolic Examples

When is the improvement maximum?

```
> diff(ImprBad(x),x);
```

$$-0.3751800137 + 0.003751800138\, x$$

```
> plot(%,x=0..100);
```



Symbolic Examples

Symbolic Example 6: Inverse problem: another case (Badajoz)

```
> ImprBad(0);
```

451.7947708

How much should the timings to Badajoz be improved in order to improve the whole timings in Spain a 5% ?

```
> solve( ImprBad(x)=(95/100)*ImprBad(0) , x);
```

$99.99999997 + 45.18937815\, I,\ 99.99999997 - 45.18937815\, I$

(it is not possible --due to the much lower number of passengers in this route).

FIN

```
>
```

Conclusions

The approach to the estimation of radial railway network improvement using **isochrone circle graphs** due to these authors was very innovative, but its implementation using a dynamic geometry system, although comfortable, had some drawbacks. Using a computer algebra system instead allows to bypass these drawbacks. Moreover, the possibility of computer algebra systems to perform symbolic computations enhance the possibilities of the approach.

Estimating radial railway network improvement with a CAS

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