An interactive evolutionary multiobjective optimization method based on the WASF-GA algorithm

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Solving a Multiobjective Optimization (MOP) problem

Two points of view:

✓ Multiple Criteria Decision Making (MCDM): helping the decision maker (DM) to find his/her most preferred solution.

✓ Evolutionary Multiobjective Optimization (EMO): generating a set of well-distributed Pareto optimal solutions approximating the whole (unknown) Pareto front.
The Weighting Achievement Scalarizing Function Genetic Algorithm (WASF-GA)

For a multiobjective optimization problem:

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\begin{align*}
\text{minimize} & \quad \{f_1(x), f_2(x), \ldots, f_k(x)\} \\
\text{subject to} & \quad x \in S.
\end{align*}
\]

the DM gives a reference point \( q = (q_1, \ldots, q_k) \).

Where are the probably most interesting nondominated solutions for this \( q \)?

\[ \implies \text{Region of interest of the Pareto front from } q. \]

How can we generate these nondominated solutions?

\[ \implies \text{WASF-GA is based on:} \]

- An achievement scalarizing function (ASF).
- The classification of the individuals into several fronts at each generation.

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Wierzbicki’s achievement scalarizing function

General Formulation

\[ s(q, f(x), \mu) = \max_{i=1, \ldots, k} \{ \mu_i (f_i(x) - q_i) \} + \rho \sum_{i=1}^{k} (f_i(x) - q_i) , \]

where \( \mu = (\mu_1, \ldots, \mu_k) \) is a vector of positive weights (\( \mu_i \in (0, 1) \) for every \( i = 1, \ldots, k \)) and \( \rho > 0 \) is the so-called augmentation coefficient.

Achievable reference point

Unachievable reference point
Wierzbicki’s achievement scalarizing function

Any Pareto optimal solution in the Region of interest from \( q \) can be obtained by minimizing \( s(q, f(x), \mu) \) over \( S \) and varying \( \mu \) in the weight vector space \((0, 1) \times \ldots k \times (0, 1)\).
Classification of the individuals into several fronts

Let $W$ be a set of $N_\mu$ vectors of weights as evenly distributed as possible in $(0, 1) \times \ldots \times (0, 1)$:

$$W = \{ \mu^j = (\mu^j_1, \ldots, \mu^j_k), \mu^j_i \in (0, 1) \text{ for every } i = 1, \ldots, k, j = 1, \ldots, N_\mu \}$$

- Problems with 2 objectives ⇒ Generating $N_\mu$ evenly distributed weight vectors is easy.
- Problems with $k \geq 3$ objectives ⇒ We will generate a sample of $N_\mu$ weight vectors which represent $(0, 1) \times \ldots \times (0, 1)$ as evenly as possible.

The classification of the individuals into the different fronts is done according to the values that every individual takes on the ASF for the $N_\mu$ weight vectors in $W$. 

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The classification of the individuals into the different fronts is done according to the values that every individual takes on the ASF for the $N_\mu$ weight vectors in $W$. 
At each generation, solutions with the best values of the ASF in the $N_{\mu}$ weight vectors and with $q$ as reference point are emphasized.

- Each front is formed by $N_{\mu}$ solutions ($N_{\mu} \leq N$).
- Nondominated solutions are preferred over dominated ones:
  
  If $x$ dominates $\bar{x}$ $\Rightarrow$ $s(q, f(x), \mu) < s(q, f(\bar{x}), \mu)$, for every weight vector $\mu$ $\Rightarrow x$ belongs to a lower level front than $\bar{x}$.

- Output: $N_{\mu}$ solutions (first front of the last generation), which approximate the region of interest from $q$. 

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Motivation

- An interactive method can very useful to solve a multiobjective optimization problem.
- There are many interactive MCDM methods but however only few interactive EMO algorithms in the literature.
- Many multiobjective optimization problems cannot solve by means of MCDM techniques.
- An interactive method based on EMO algorithms is able to solve many kinds of multiobjective optimization problems.
- The WASF-GA’s features allow us to build an interactive method in an easy way.
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Main ideas

• It is based on the WASF-GA algorithm.
• Given a reference point $q$, the DM decides how many solutions ($N_S$) wants to obtain and to see for these reference values. For default, $N_S$ can be equal to $2^k$.
• $N_S$ weight vectors are generated, which are dispersed between them and evenly distributed as much as possible.
• $N_S$ nondominated solutions are generated in the region of interest by the WASF-GA algorithm.
• At each iteration, Interactive WASF-GA can be very fast since that only few weight vectors are considered ($N_S$).
• The final population of one iteration is used as initial population in the following one.
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Interactive WASF-GA

Motivation
Main ideas
Computational implementation

The End

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It is based on the WASF-GA algorithm.

Given some reference levels by the DM, several nondominated solutions are generated in a region of interest.

A number of weight vectors equals to the number of solutions to be shown to the DM must be considered.

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THANK YOU VERY MUCH FOR YOUR ATTENTION!